Fluid mechanics & machinery

Unit-I Fluid properties and flow characteristics

(2 mark or part B 8 mark)

Properties of fluid

Fluid Mechanics is highly relevant to our daily life. we live in the world full of fluids. Fluid mechanics covers many areas such as oceanography, areodynamics, biomechanics, hydraulics, mechanical Engineering & civil Engg etc.

It does not explain only scientific phenomena but also involved in industrial applications.

Fluid

The main difference b/w fluid and solid is their behaviour when shear force acting on them. A certain amount of displacement is found when a shear force is applied to a solid element.

Which one of the element give low resistance to flow, that is called fluid.

Properties

1. Density (ρ)

It is the mass/unit of volume in fluid.

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \quad kg/m^3$$

Air = 1.204 kg/m^3

water = 998 kg/m^3

Hg = 13550 kg/m^3.

2. Relative density (R)

S.G = density of substance/density of water $= \frac{\rho_1}{\rho_w}$.

3. Sp. volume : (V_s) It is a volume of fluid occupied by unit of mass.

$$V_s = \frac{\text{volume}}{\text{mass}} = \frac{V}{m} \quad m^3/kg \qquad V_s = \frac{1}{\rho}.$$

4. Specific weight (V_w) : It is weight per unit of volume.

$$V_w = \frac{\text{weight}}{\text{Volume}} = \frac{W}{V} \quad kN/m^3. = \frac{m \times g}{V} = \rho \times g.$$

5. Compressibility:

When fluid are pressurized, the volume (Total) V is changed. The amount of volume is the compressibility of fluids.

Bulk modulus $E_v = \frac{-dp}{dv/v}$.

6. Cohesion:

It's a intermolecular attraction b/w molecules of the same fluid.

7. Adhesion: It meas attraction b/w the molecules of different liquids and the molecules of a solid boundray surface in contact with fluid.

8. Viscosity (μ): It is the frictional resistance of fluid

$$\frac{\text{Force} \times \text{time}}{\text{length}^2} = N\text{-}s/m^2.$$

9. Sp. gravity (s)

$$S_g = \frac{\text{sp. weight of liquid}}{\text{Sp. weight of water } (9810\ N/m^3)}.$$

10. Capillarity: It is a phenomenon by which a liquid rises into a thin glass tube above or below its general level.

11. Surface Tension: N/m. (σ)

Adhesion defined as the force of attraction b/w the molecules of two different liquids (liquid & boundary).

It is a chusive forces b/w liquid & molecules.

12. Temperature:

It is defined as a measure of velocity of fluid particle

Fluid mechanics :- It is study about the characteristics of fluid and behaviour in different conditions.

Types:

① Fluid statics :- It is study about the mechanics of fluids at Rest.

② . Fluid dynamics :- It is study of mechanics of fluid in motion.

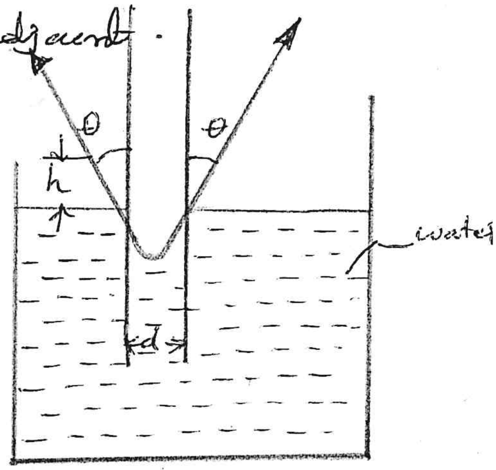
③ Fluid kinematics :- It is study about velocity, acc. in flow.

Pascal's law:
It is the resistance offered to the movement of one layer of fluid by another adjacent layer of fluid.

Types of fluid:

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-newtonian fluid.
5.

Capillarity (derive):- capillarity has a phenomenon of rise or fall of liquid surface relative to the adjacent.



d - diameter of glass tube (d).
w - sp. weight of liquid.
σ - surface Tension.

Force acting in vertical direction

$$\sigma \times \pi d \times \cos\theta \quad \text{—— ①}$$

$$\text{gravity force} = \frac{\pi}{4} \times d^2 \times h \times w \quad \text{—— ②}$$

eq ① & ②

$$\pi \cdot d \cdot \sigma \cdot \cos\theta = \frac{\pi}{4} \cdot d^2 \times h \times w$$

[The weight of liquid column in 'h' .T)
= Area of tube $\times h \times$ sp. weight].

$$h = \frac{4\sigma \times \cos\theta}{d \cdot w}$$

Here $\theta_{water} = 0°$
$\theta_{Hg} = 140°$

Viscosity of fluids (N-s/m²)

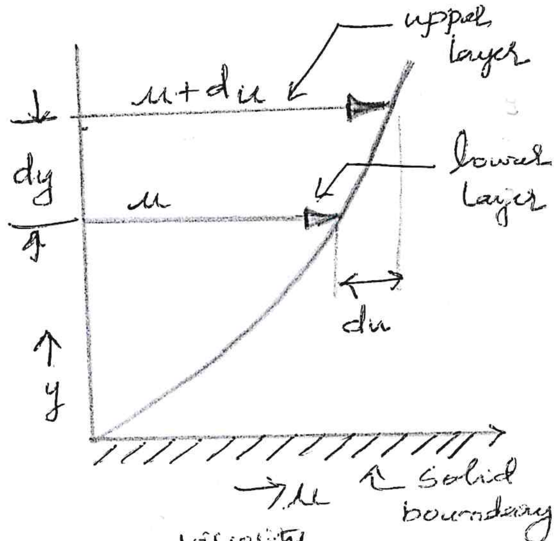
We consider two layers of fluid at a distance 'dy' apart

change of velocity $\tau \propto \frac{du}{dy}$

$$\tau = \mu \cdot \frac{du}{dy}$$

1 poise $= \frac{1}{10}$ N-s/m²

$$\mu = \frac{\tau}{du/dy}$$



Kinematic viscosity [m²/sec]

It is the ratio of fluid viscosity & density.

$$\nu = \frac{\mu}{\rho} = \frac{\text{viscosity}}{\text{Density}}$$

Newton's Law of viscosity: The law states that the shear stress (τ) on a fluid Element layer is directly proportional to the rate of shear strain. $\tau = \mu \cdot \frac{du}{dy}$.

Fluid Statics:

Fluid statics or hydrostatics is the branch of fluid mechanics that studies the condition of the equilibrium of a floating body and submerged body.

Fluid statics is the branch of fluid mechanics that studies incompressible fluid at rest. It is encompasses the study of the conditions under which fluids are at rest in stable equilibrium as opposed to fluid dynamics, the study of fluids in motion.

Pressure measurements

Pressure of fluid: (intensity of pressure)

$$\text{pressure } (P) = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{w h A}{A} = wh.$$

It may be defined as the force exerted on a unit area

1. Atmospheric pressure (Patm): It is the pr. exerted by the air on the atm.

   Atm. pr = 1.01325 bar.

2. gauge pressure: (Pg): Above value of atm. pressure.

3. vac. pressure: below the atmospheric pressure.

4. Absolute Pressure: The pressure measured from the absolute zero pressure is called abs. pr.

$P_{abs} = P_{atm} + P_g$.

$P_{abs} = P_{atm}$. ; Pr. head $p = \frac{P}{w}$.

Pressure measurements by manometers

Manometers

- Simple manometer
  - i) piezo meter
  - ii) U tube manometer
  - iii) single column manometer
- differential manometer
  - 1) U tube diff. manometer
  - 2) Inverted U-tube manometer.

U-tube manometer

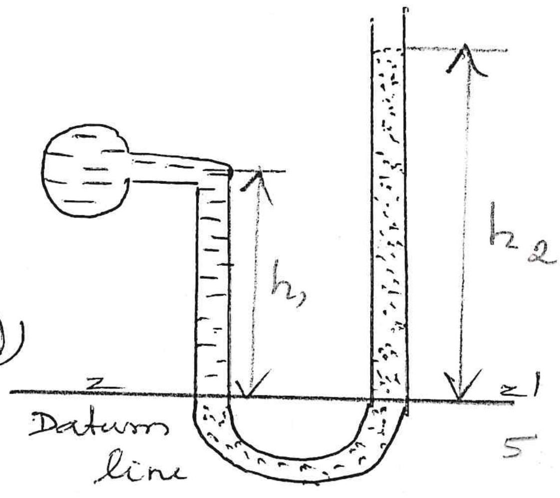
h - pr. head

h_1 - height of liquid

h_2 - height of mercury (heavy liquid)

S_1 - sp. gravity of water

S_2 - sp. gravity of Hg.

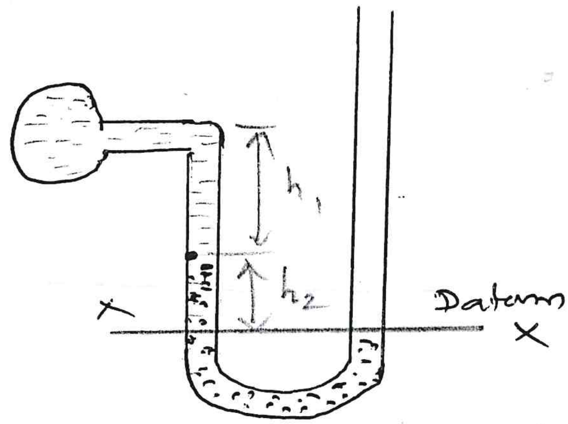


$L.H.S = R.H.S$

$p + \rho_1 \cdot g \cdot h_1 = \rho_2 \cdot g \cdot h_2$

$P = \rho_2 \cdot g \cdot h_2 - \rho_1 \cdot g \cdot h_1$ (or) $H = h_2 S_2 - h_1 S_1$

II. Vacuum Pressure:



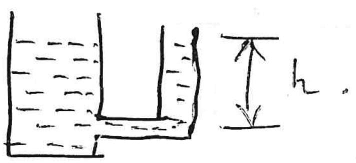
$L.H.S = R.H.S$

$H + h_1 \cdot S_1 + h_2 \cdot S_2 = 0$

$H = -h_1 S_1 - h_2 S_2$.

i $P = -(\rho_2 g h_2) + (\rho_1 g h_1)$

III. Piezo meter



$Pr. head = \frac{Pr}{w}$

$p = w \times h$

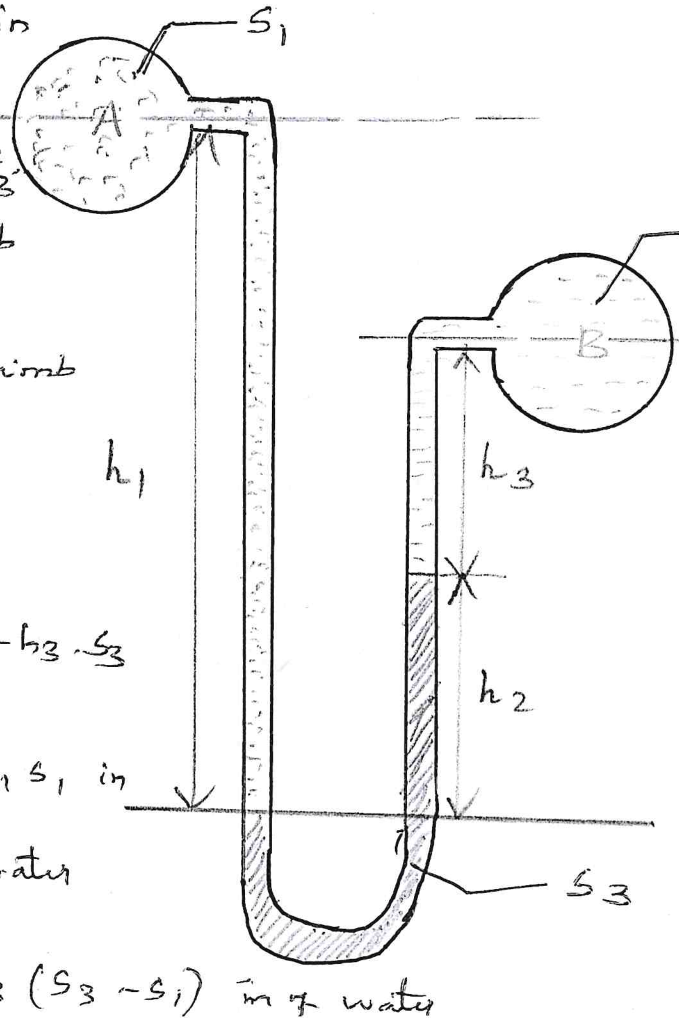
$p = \rho \times g \times h$.

IV. U Tube differential manometer

h_1 - height of liquid in pipe A

h_2 - height of heavy liquid in right limb

h_3 - height of liquid in pipe B



Pr. head in the left limb

$h_A + (h_1 \cdot S_1)$

Pr. head in the Right limb

$h_B + h_2 S_2 + h_3 S_3$

$L.H.S = R.H.S$

$h_A + h_1 \cdot S_1 = h_B + h_2 S_2 + h_3 \cdot S_3$

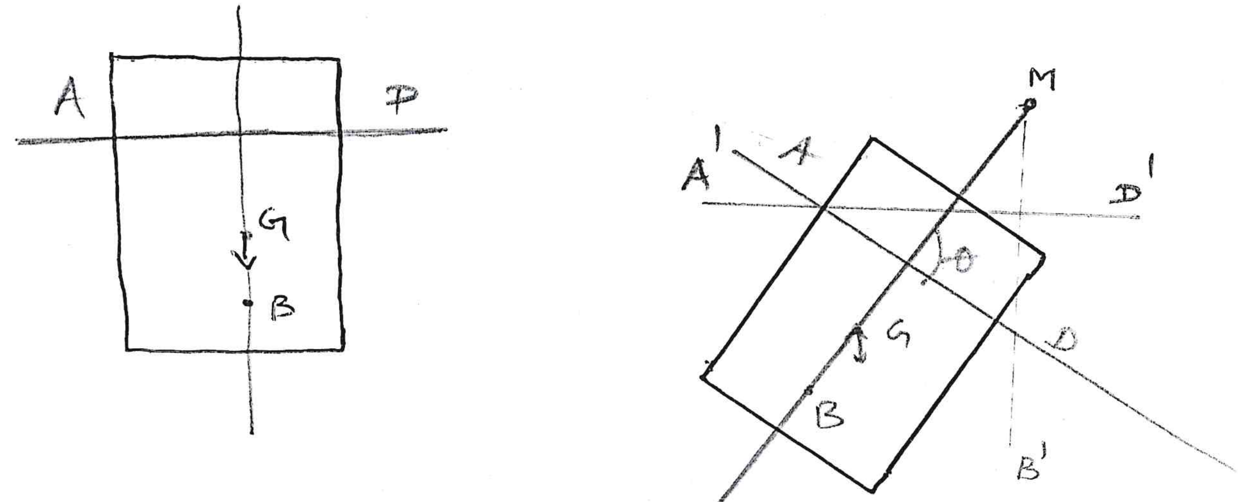
$h_A - h_B = h_2 S_2 + h_3 S_3 - h_1 S_1$ in m of water

If same hight

$h_A - h_B = h_2 (S_2 - S_1) + h_3 (S_3 - S_1)$ m of water

Buoyancy and floatation

when a body is immersed in a fluid either fully or partially, it is subjected to an upward force. This upward force will tend to lift the body up. This is tendency for a immersed body to the lifted up in the fluid due to upward force opposite to the action of gravity known as buoyancy.



when the body is tilted, the portion of the body immersed on the right hand side decreases.

This line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at point 'M' which is known as meta centre.

problem ① A cylinderical vessel of 130 mm diameter is filled with a liquid upto depth of 80 mm Subsequently 100 mm diameter solid cylinder of height 90 mm and weighting 4×10^{-3} N and sp. weight 8.5 kN/m^3 is immersed into the liquid contained in the vessel. Determine the level at which the solid cylinder will float.

Given: Dia. of vessel $= 130\,mm = 0.13\,m$

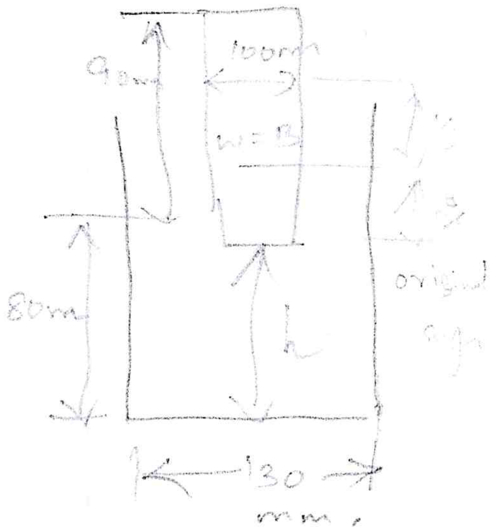
Sp. weight of liquid $= 8.5\,kN/m^3$

Height of liquid in vessel $= 80\,mm$
$= 0.08\,m$

Diamter of solid cylinder $= 100\,mm$
$= 0.1\,m$

weight of solid cylinder $= 4 \times 10^{-3}\,N$

Height of solid cylinder $= 90\,mm$
$= 0.09\,m$



x = distance of solid cylinder below original liquid surface

y = distance of the liquid rise above the original surface.

Solution:

Pressing area

Volume of liquid depressed = volume of liquid rise

$$\frac{\pi}{4} \times (0.1^2) \times \overset{L}{x} = \left[\frac{\pi}{4}(0.13)^2 - \frac{\pi}{4}(0.1^2)\right] \times y$$

$$x = 0.69\,y \quad \text{—— (1)}$$

weight of solid cylinder = Boyance force

$\frac{W}{V} = w$ = weight of liquid displaced by submerge.

4×10^{-3} = weight of liquid $\times$ Volume

$$4 \times 10^{-3} = 8.5 \times \left[\frac{\pi}{4} \times (0.1^2) \times (x+y)\right]$$

$$x + y = 0.06 \quad \text{—— (2)}$$

Eq (1) & (2)

$x + y = 0.06$

$0.69y + y = 0.06$

$y = 0.035$

$x = 0.0224\,m$

floating level $80 - x$

$= 80 - x$

$= 0.08 - 0.0224$

Flow characteristics:

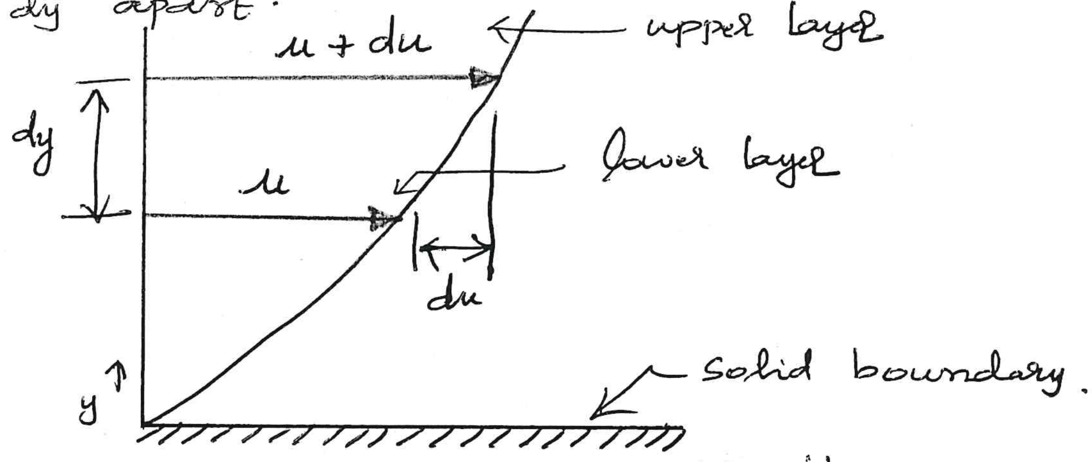
Newton's law of viscosity:

This law states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau = \mu \cdot \frac{du}{dy}.$$

Viscosity of fluids: ($N\text{-}s/m^2$)

We consider two layers of fluid at a distance 'dy' apart.



$\mu \rightarrow$ viscosity

change of velocity

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\mu = \tau \Big/ \frac{du}{dy}$$

1 poise = $\frac{1}{10}$ $N\text{-}s/m^2$.

Kinematic viscosity (m^2/sec)

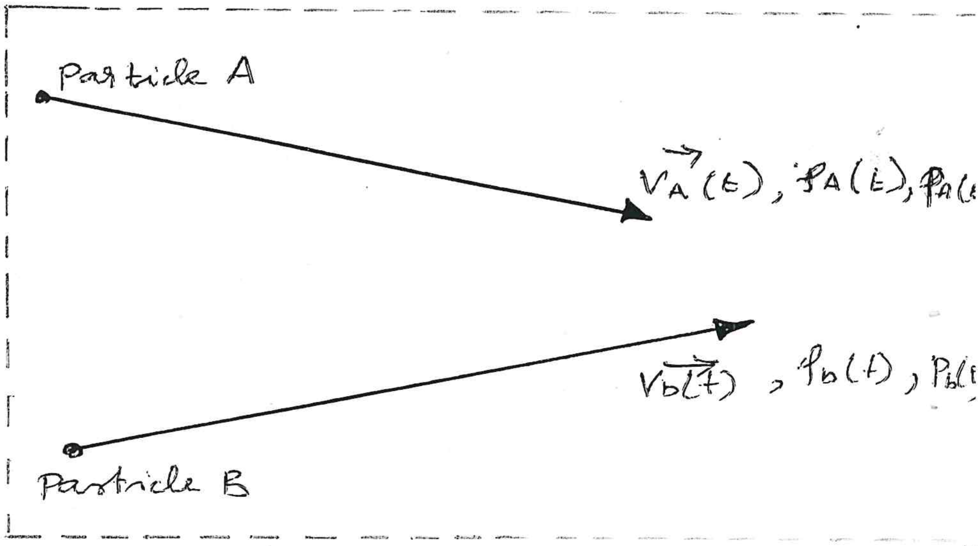
$$\nu = \frac{\mu}{\rho} = \frac{\text{Viscosity (or) dynamic viscosity}}{\text{Density of fluid}}.$$

Eulerian & Lagrangian approach

In the Lagrangian description of fluid flow, the observer follows an individual fluid particle in the fluid motion as it moves through space and time.

The fluid flow properties are determined by tracking the motion and properties of the particles as they move in time.

Let consider particles A and B in the fluid flow as shown in figure. Position vectors and velocity vectors are shown at one instant of time for each of these marked particles.



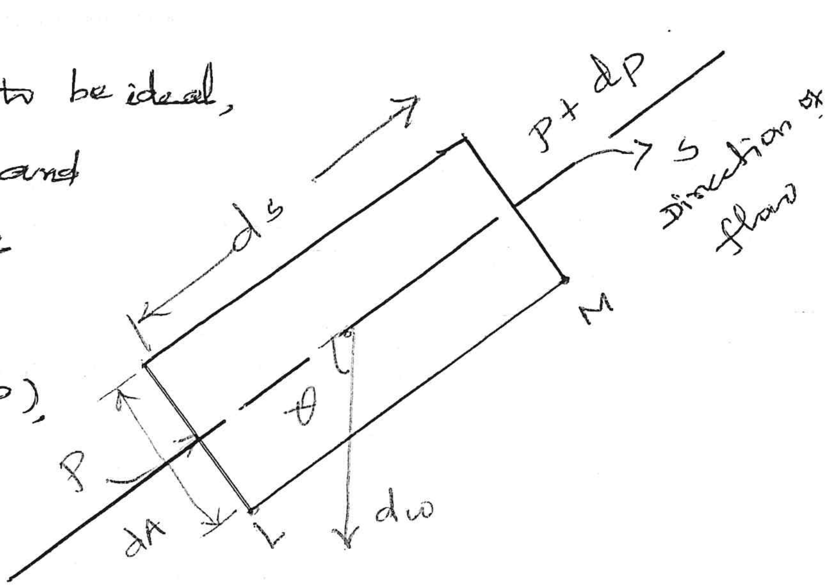
The flow velocity, density and pressure of particle A is represented as $\vec{V_A}(t)$, $\rho_A(t)$ and $P_A(t)$

In the Eulerian description of fluid flow, individual fluid particles are not identified. In this, the physical laws such as Newton's laws and the laws of conservation of mass and energy cannot be apply directly to the particles as in a Lagrangian Description.

Hence, some translation or reformulation of these laws is required for use.

Euler equation:

when the flow is assumed to be ideal, the viscous force (F_v) is zero and the equation of motion are Euler's equation.



Consider force (F), pressure (P), which flow in 'X' direction.

cross section of cylinder element dA & ds

Here, P – Pressure act on element 1 (L)

p+dp – pressure act on element (M)

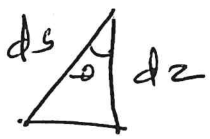
V – Velocity of fluid element

W.K.T, Net force acting on fluid element in direction 's'

$$\text{net force} = p \cdot dA - (p+dp) \cdot dA$$
$$= p \cdot dA - p \cdot dA + dp \cdot dA$$
$$= -dp \cdot dA \quad \text{— (i)}$$

W.K.T, weight of the fluid element in the direction of flow

$$dw = -\rho \cdot g \times \underbrace{dA}_{(\text{Area})} \cdot \underbrace{ds}_{\text{length}} \times \cos\theta$$
$$= -\rho \cdot g \cdot dA \cdot ds \cdot \left(\frac{dz}{ds}\right)$$
$$= -\rho \cdot g \cdot dA \cdot dz \quad \text{— (2)}$$



Resultant force = Net force + weight →
$$= -dp \cdot dA - \rho \cdot g \cdot dA \cdot dz$$

According the Newton's second law

$$F = \text{mass} \times \text{Acc}$$

$$a = \frac{dv}{dt}$$

$$(-dp \cdot dA) - (\rho g \cdot dA \cdot dz) = \rho \cdot d \cdot A \cdot ds \times \frac{dv}{dt}$$

$\times \div$ by ds $\quad a = \frac{dv}{ds} \cdot \frac{ds}{dt}$

$$= \rho \cdot d \cdot A \cdot ds \times v \cdot \frac{dv}{ds}.$$

$= v \cdot dV/ds$.

$$\div \rho \cdot dA \Rightarrow -\frac{dp \cdot dA}{\rho \cdot dA} - \frac{\rho \cdot g \cdot dA \cdot dz}{\rho \cdot dA} = \frac{\cancel{\rho \cdot dA} \cdot ds \times v \cdot dv/ds}{\cancel{\rho \cdot dA}}$$

$$-\frac{dp}{\rho} - g \cdot dz = v \cdot dv.$$

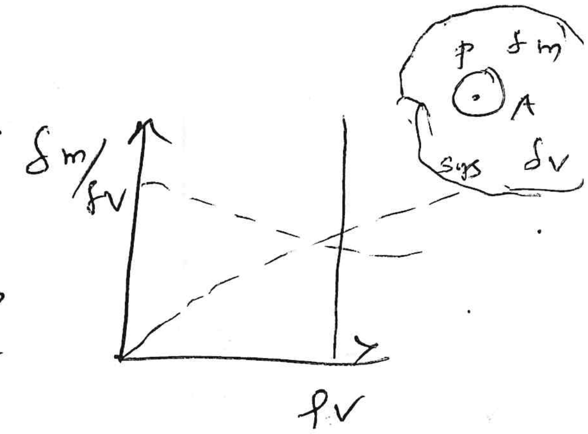
$$\frac{dp}{\rho} + g \cdot dz + v \cdot dv = 0 \quad \text{equation//.}$$

Concept of control volume & system

continum concept of system:

i) In liquids, the molecules are closely spaced which create strong inter molecules cohesive force. so liquids get a continuous mass.

ii) The overall motion of liquid will be continuous medium. That is called continuum.



A point of mass δm and occuping a volume of δv in fig.

The point give continuous motion suppose at point 'A' non contain any molecules. V↓ small

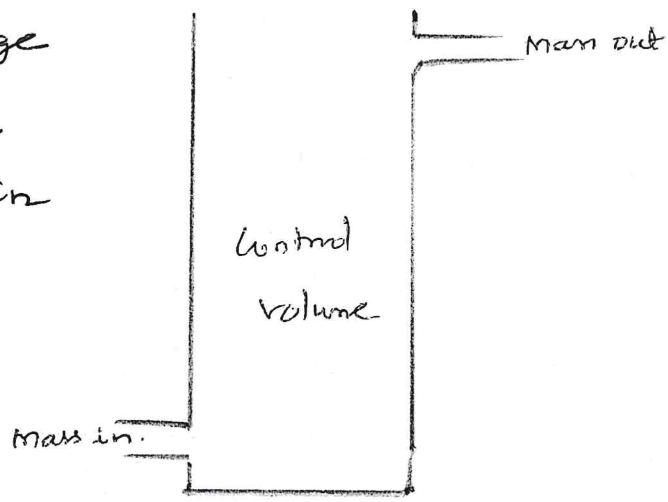
$\rho = 0$

$\rho = \frac{m}{v}$.

If the point has same molecules, the density may be high/low the maximum density is called critical volume. Here the density is constant. $\rho = dv \rightarrow \delta v_c \left(\frac{dm}{\delta v}\right)$.

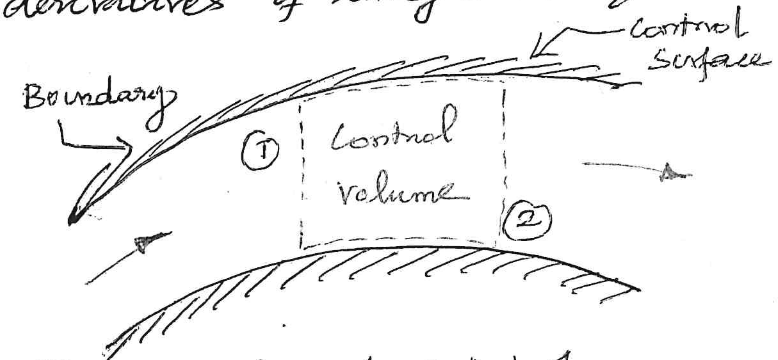
control volume:

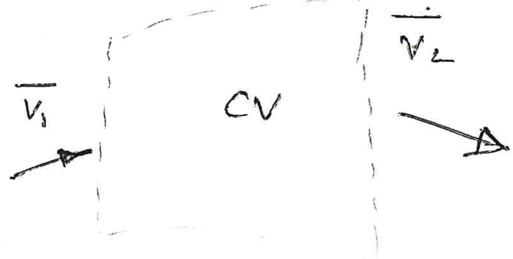
It is a specified large number of fluid and thermal devices have mass of flow in and out of a system is called control volume.



Reynold's Transportation Theorem

Reynolds transport theorem is also known as Leibniz – Reynolds transport theorem. It is used to compute derivatives of integrated quantities.





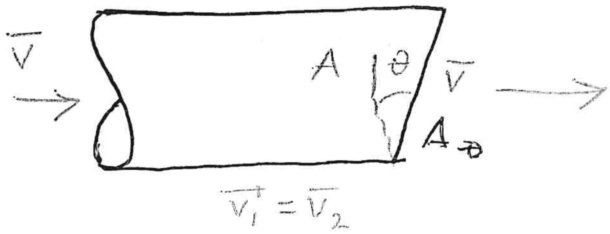
CV - control volume
CS - control surface.

$\bar{V}_1$ - velocity at point 1
$\bar{V}_2$ - velocity at point 2

In continuity equation $Q = AV \Rightarrow$ & $m = \rho \cdot A \cdot V$

So mass flow rate $m = \rho A_\theta V$

$$m = \rho |V| |A| \cos\theta$$
$$= \rho (\bar{V} \cdot \bar{A})$$



Similarly cont. equation $Q = |V| \cdot |A|$

Net flow rate in the control volume

$$Q_{net} = \sum_{CS} \bar{V} \cdot \bar{A}$$

Mass flow rate $\bar{m}_{net} = \sum_{CS} \rho \bar{V} \cdot \bar{A}$

Extensive property of mass

$$B = \sum_{CS} b \rho \bar{V} \cdot \bar{A}$$

where $b \Rightarrow b = \frac{B}{m}$

So Extensive property of mass $B = \int_{CS} b \rho \bar{V} \cdot d\bar{A}$

So the derivative form of extensive property of mass

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{t+\Delta t} - B_t}{\Delta t} \right] \Rightarrow \lim_{\Delta t \to 0} \left[\frac{(B_2 + B)_{t+\Delta t} - (B_1 + B)}{\Delta t} \right.$$

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \left[\frac{B_{2t+\Delta t} - B_{2t}}{\Delta t} \right] + \lim_{\Delta t \to 0} \left[\frac{B_{3t+\Delta t} - B_{1t}}{\Delta t} \right]$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} b \rho \, dv + \sum_{CS} b \rho \bar{V} \bar{A}$$

eqn

Continuity equation (3-dimensions)

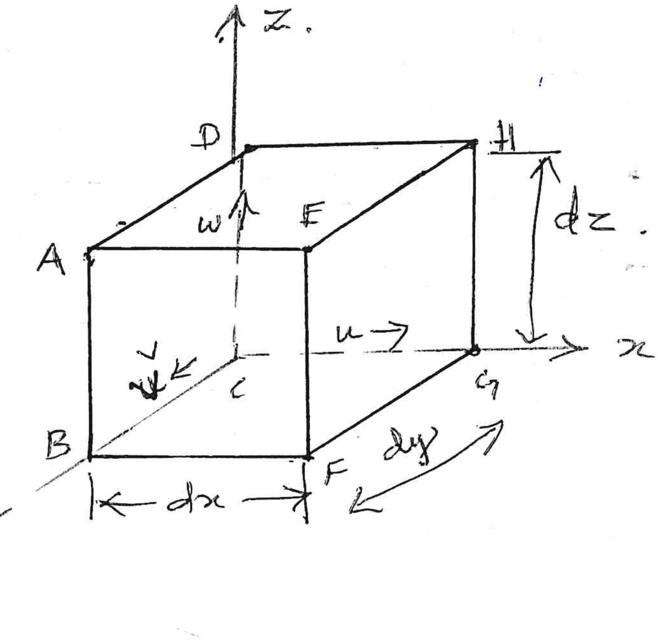
Consider a fluid element

dx, dy & dz are length

let u - inlet velocity in 'x'

v - inlet velocity in 'y'

w - inlet velocity in 'z'



In ABCD, mass of fluid

$= \rho \times$ Velocity in 'x' $\times$ Area of ABCD.

$= \rho \times u \times dy \times dz$

In EFGH, mass of fluid

$= \rho . u . dy . dz + \frac{\partial}{\partial x}(\rho u \, dy \, dz) dx$

Gain mass in 'x' direction (mass through ABCD − mass through EFGH)

$= \rho . u . dy . dz - (\rho . u . dy . dz + \frac{\partial}{\partial x}(\rho u . dy . dz) dx$

$= -\frac{\partial}{\partial x} . (\rho u . dy . dz) . dx \Rightarrow -\frac{\partial}{\partial x}(\rho . u) . dx . dy . dz .$

III^{ly} the net gain in 'y' direction

$= -\frac{\partial}{\partial y}(\rho . v) . dx . dy . dz$

III^{ly} the net gain in 'z' direction

$= -\frac{\partial}{\partial z}(\rho . w) . dx . dy . dz$

So, total masses $= -\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho w)\right] dx . dy . dz$ — ①

mass not destroyed, mass increase with respect to time.

$\frac{\partial}{\partial t}(\rho . dx . dy . dz)$ — ②

Equate ① & ②

$-\left[\frac{\partial}{\partial x}(\rho . u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho . w)\right] \cancel{dx . dy . dz} = \frac{\partial \rho}{\partial t}(\cancel{dx . dy . dz})$

$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho . u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho . w) = 0$

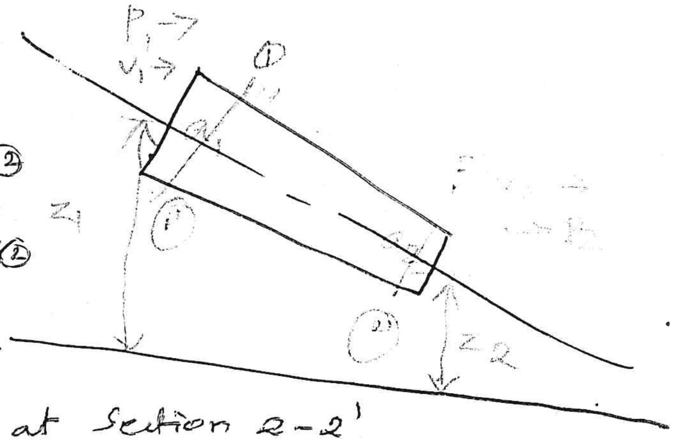
$\frac{\partial}{\partial x}(\rho . u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho . w) = 0$ (for steady flow $\frac{\partial \rho}{\partial t} = 0$)

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (for incompressible flow $\rho = c$)

eqn //.

Energy Equation :- Bernolius Equation, theorem

In an ideal (or) incompressible fluid when the flow is steady and continuous, the sum of pressure Energy, kinetic energy and potential energy is constant along the stream line.



a_1 & a_2 - area of pipe at sect ① & ②

V_1 & V_2 - velocity of fluid at sect ① & ②

p_1 & p_2 - pressure of fluid at sect ① & ②

Z_1 & Z_2 - datum line in ① & ②

Volume at section 1-1' = volume at section 2-2'

$$a_1 \cdot dL_1 = a_2 \cdot dl_2$$

W.K.T,

$$w = \frac{W}{V} \; ; \; V = \frac{W}{w}$$

$$a_1 \cdot dL_1 = a_2 \cdot dl_2 = \frac{W}{w} \quad \text{--- (1)}$$

I. Gain of kinetic Energy - K.E ↑

K.E will be in $V_2 > V_1$

Gain K.E = K.E at ② - K.E at ①

$$= W \cdot \frac{V_2^2}{2g} - W \cdot \frac{V_1^2}{2g} = W\left[\frac{V_2^2 - V_1^2}{2g}\right] \quad \text{--- (2)}$$

II. Loss of Potential Energy . P.E ↓

Loss of P.E = $WZ_1 - WZ_2$ $\quad (Z_1 > Z_2)$ --- (3)

III. Workdone

Net workdone $= P_1 \cdot a_1 \cdot dL_1 - P_2 \cdot a_2 \cdot dl_2$

$$= (P_1 - P_2) \cdot a_1 \cdot dl_1$$

$$= (P_1 - P_2) \cdot \frac{W}{w} \quad \text{--- (4)}$$

By Law of conservation,

Total Energy at 1-1' = Total Energy 2-2'

Gain of K.E = Loss of P.E + workdone.

$$W\left[\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right] = W\left[Z_1 - Z_2\right] + \frac{W}{w}\left(P_1 - P_2\right)$$

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{V_2^2}{2g} + \frac{P_2}{w}.$$

Momentum equations ($m \times v$)

To find out force produced on a solid body by the action of flowing fluid

Principle: The time rate of change of momentum is proportional to the impressed force and takes place in the direction in which force acts.

According the second law of motion

$$F = mass \times ac.$$

$$= m \times \frac{dv}{dt} \quad ; \quad F = \frac{\partial (mv)}{dt}$$

Impulse applied force = change in momentum

$$F \cdot dt = d(mv)$$

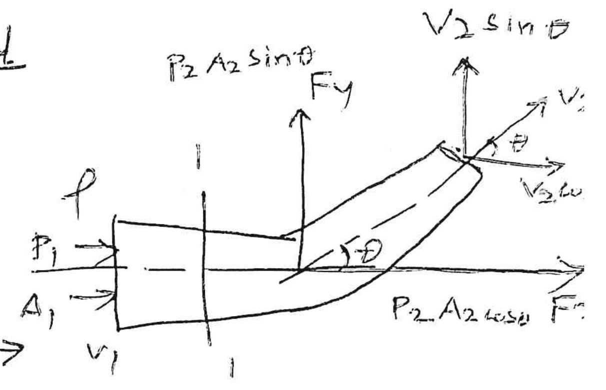
This is impulse momentum equation

Force exerted on a pipe Bend

Here, P_2, P_1 – Pressure at section 1, 2

V_1, V_2 – velocity at section 1, 2

A_2, A_1 – Area of section 1, 2



The momentum equation, In x →

Net force acting on fluid in direction x = Rate of change of momentum in x direction

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = \rho \cdot Q (V_2 \cos\theta - V_1).$$

$$F_x = \rho \cdot Q (V_1 - V_2 \cos\theta) - P_2 A_2 \cos\theta + P_1 A_1$$

III^{ly}, horizontal direction, the momentum eqn in y direction

$$(0 - P_2 A_2 \sin\theta) + F_y = \rho \cdot Q \cdot (V_2 \sin\theta - 0).$$

$$F_y = \rho \cdot Q \, V_2 \sin\theta + P_2 A_2 \sin\theta.$$

Resultant force

$$R = F_R = \sqrt{F_x^2 + F_y^2} \quad \text{eqn.}$$

Applications of Energy Equation:

limitations:

1. The flow is ideal
2. The flow is steady
3. The flow is incompressible
4. The flow is irrotational.

so, equation for real fluid

$$z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g} + h_L$$

Applications: – To measure the discharge value

1. Venturimeter
2. Orifice meter
3. Pitot tube
4. Roto meter

1 Orifice-meter

Actual Discharge $Q_t = C_d \cdot \frac{a_1 \cdot a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$

a_1 – area of pipe at section 1

a_2 – area of pipe at Section 2

P_1 – Inlet pressure

P_2 – Exit Pressure

V_1 – velocity of fluid at section 1

V_2 – velocity of fluid at Section 2

h – x manometer reading in 'm'

C_d – co-efficient of discharge of fluid

Q_t – Theoretical discharge.

Q_a – Actual discharge.

Pitot tube:

To measure the velocity of flow

$$V = \sqrt{2gh}$$

V - velocity of fluid through pipe

h - height of liquid in pitot tube above the free surface of water

venturi meter.

$$\text{Discharge } Q_{act} = Cd \cdot \frac{a_1 \cdot a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}.$$

a_1 - area of inlet section 1

P_1 - pressure at section 1

V_1 - velocity at section 1

a_2 - area of throat section 2.

P_2 - pressure at section 2.

V_2 - velocity at section 2

Q_{act} - Actual discharge.

Inclined venturimeter with Differential U-tube manometer

$$h = \frac{P_1}{\rho \cdot g} + z_1 - \frac{P_2}{\rho g} + z_2$$

$$h = x \left[1 - \frac{S_L}{S_0} \right]$$

Classification of fluids

I. Ideal fluid:

- It is incompressible ; zero velocity
- shear force is zero when the fluid is in motion

II. Real fluid:

- It is compressible ; has viscosity
- Some resistance is always offered by the fluid when it is motion

III. Newtonian fluid:

In newtonian fluids, a linear relationship exists b/w the magnitudes of shear stress τ and the resulting rate of deformation.

Example: water, kerosene

IV. Non-Newtonian fluid

In non-newtonian fluids, there is a non-linear relation between the magnitude of the applied shear stress and the rate of deformation.

Problem (1) Calculate the specific weight, mass density, specific volume and specific gravity of 2 litre of petrol weights 13 N.

Given:

Volume = 2 lit = $2 \times 10^{-3}\ m^3$

weight = 13 N.

Solution:

Sp. weight $w = \frac{W}{V} = \frac{13}{2 \times 10^{-3}} = 6500\ N/m^3$

2. Mass density = $\rho = \frac{W}{g} = \frac{6500}{9.81} = 662.59 \text{ kg/m}^3$

3. Sp. volume $v = \frac{1}{\rho} = \frac{1}{662.59} = 1.51 \times 10^{-3} \text{ m}^3/\text{kg}$.

Problem 2

Determine the capillarity rise in a tube of 5 mm diameter, when immersed in i) water and ii) mercury. The temperature of water and mercury is 20°C and the surface tension of water and mercury at 20°C in contact with air are 0.07358 N/m and 0.51 N/m and the density of water and mercury are 998 kg/m³ and 13550 kg/m³. The contact angle for water is 0° and for mercury is 130°.

Given: Temp. of water $T_w = T_m = 20°C$

Surface Tension $\sigma_w = 0.07358$ N/m

$\sigma_m = 0.51$ N/m.

density

$\rho_w = 998 \text{ kg/m}^3$; $\rho_m = 135550 \text{ kg/m}^3$.

Contact angle $\theta_w = 0°$ & $\theta_{Hg} = 130°$.

Solution:

i) capillary rise for water

$$h = \frac{4\sigma\cos\theta}{wd}$$

$$w = \frac{W}{V} = \rho \times g$$

$$= \frac{4 \times 0.07358 \times 1}{(\underset{\rho}{998} \times \underset{g}{9.81}) \times 5 \times 10^{-3}}$$

$$= 6.01 \times 10^{-3}$$

$h = 0.006$ m

ii) capillary rise for H_g

$$h = \frac{4\sigma\cos\theta}{w.d}$$

$$= \frac{4 \times 0.51 \times \cos 130°}{(13550 \times 9.81) \times 0.0}$$

$h = -1.97 \times 10^{-3}$ m

Reynold's Experiment:

The ratio of the inertia and viscous force is called Reynolds number

$$\text{Reynold number } Re = \frac{\text{Inertia force}}{\text{viscous force}}$$

$$\begin{aligned}
\text{Inertia force} &= \text{mass} \times \text{acc} \\
&= \rho \times v \times a \\
&= \rho \times L^3 \times \left(\frac{L}{T^2}\right) \\
&= \rho \cdot \frac{L^4}{T^2} \\
&= \rho \cdot L^2 \cdot \left(\frac{L}{T}\right)^2 \\
&= \rho \cdot L^2 \cdot v^2
\end{aligned}$$

$$\begin{aligned}
\text{viscous force} &= \text{shear Stress} \times \text{Area} \\
&= \mu \cdot \frac{dv}{dy} \times L^2 \\
&= \mu \times \frac{v}{L} \times L^2 \\
&= \mu \cdot v \cdot L
\end{aligned}$$

$$\text{Reynolds number } Re = \frac{\rho \cdot L^2 \cdot v^2}{\mu \cdot v \cdot L} = \frac{\rho \cdot v \cdot L}{\mu}$$

$$= \frac{v \cdot D}{\nu}$$

$$\text{kin. viscosity } \nu = \frac{\mu}{\rho}$$

Laminar flow through circular pipes

Hagen Poiseuille equation

Consider a horizontal pipe of radious R. The viscous fluid is flowing from left to right of the pipe.

$\bar{u}$ = avg. velocity

r = fluid element radious

Δx – length of fluid element

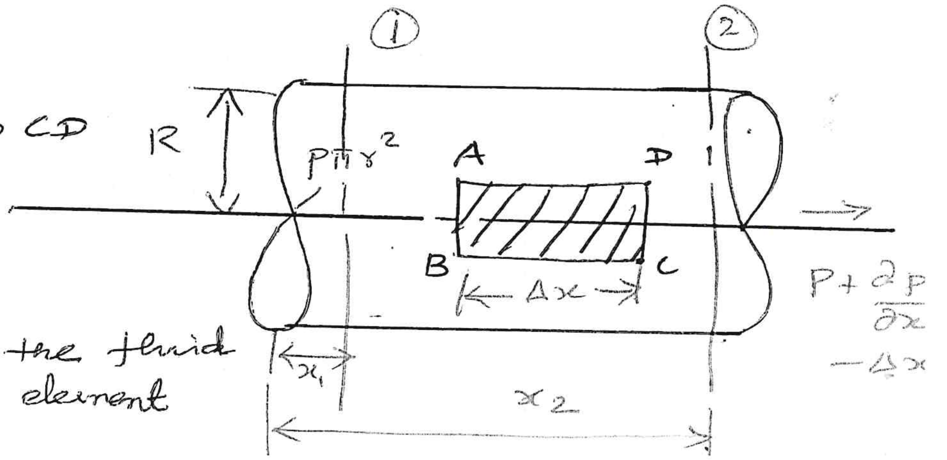
μ – viscosity of fluid

① The pressure force act on AB

$$p \times \pi r^2$$

② The pr. force act on CD

$$\left(p + \frac{\partial p}{\partial x} \cdot \Delta x\right) \pi \cdot r^2$$



③ The shear force on the fluid element

$$\tau \times 2\pi r \times \Delta x$$

So,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x} \Delta x\right) \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$\cancel{p\pi r^2} - \cancel{p\pi r^2} - \frac{\partial p}{\partial x} \pi r^2 \Delta x - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial p}{\partial x} \Delta x \cdot \pi r^2 - \tau \times 2\pi r \Delta x$$

$$-\frac{\partial p}{\partial x} \Delta x \pi r \,(r - 2\tau) = 0$$

$$-\frac{\partial p}{\partial x} \cdot r - 2\tau = 0$$

$$-\tau = \frac{\partial p}{\partial x} \cdot \frac{r}{2}.$$

$$\tau = \frac{-\partial p}{\partial x} \cdot r/2.$$

$$\tau_{max} = -\frac{\partial p}{\partial x} \cdot R/2$$

Darcy weisback equation – To find loss of head h_f' for Turbulent flow.

Consider a horizontal pipe having Steady flow.

A – Area of pipe

L – length of pipe

P – wetted perimeter

P_1 – pr. at section ①

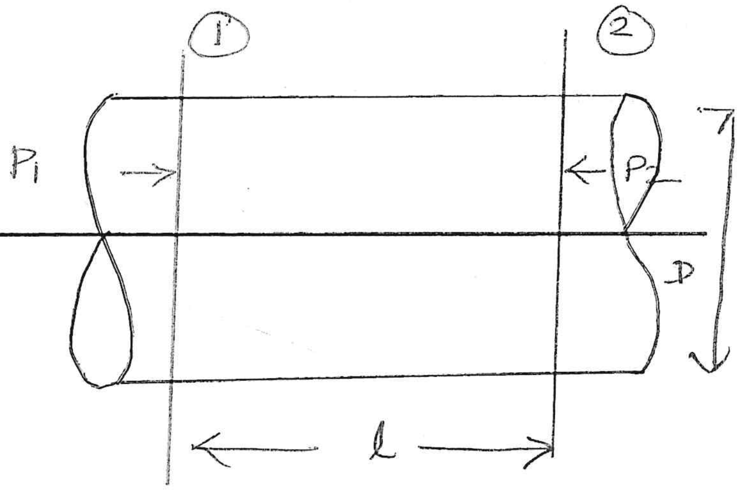
P_2 – pr. at Section ②

L – length of pipe b/w ① & ②

D – Dia. of pipe

f' – friction factor

V – avg. velocity



Force act on two sections

$$F = (P_1 - P_2) \cdot A \quad \text{—①}$$

Frictional resistance force

$$F = f' \cdot P \cdot L \cdot V^2 \text{—②}$$

Equate ① & ②

$$(P_1 - P_2) A = f' \cdot P \cdot L \cdot V^2 .$$

÷ by w

$$\left(\frac{P_1 - P_2}{w}\right) \cdot A = \frac{f'}{w} \cdot P \cdot L \cdot V^2 .$$

$$h_f = \frac{f'}{w} \cdot \left(\frac{P}{A}\right) \cdot L \cdot V^2 .$$

÷ & ⊗ by 2g

$$h_f = \frac{f' 2g}{w} \cdot \frac{P}{A} \cdot \frac{L \cdot V^2}{2g} .$$

The ratio of A/P is called Hydraulic mean depth $m = \frac{A}{P}$

$$h_f = \frac{2 \cdot g f'}{w} \cdot \frac{P}{A} \cdot \frac{LV^2}{2g}$$

$$= \frac{2 \cdot g f'}{w} \cdot \frac{1}{m} \cdot \frac{LV^2}{2g} \qquad \frac{f'}{\rho} = f/2 .$$

$$= f \cdot \frac{L}{M} \cdot \frac{V^2}{2g} .$$

Here

$$m = \frac{A}{P}$$

$$h_f = f \cdot \frac{L}{D/4} \cdot \frac{V^2}{2g} = \frac{4 \cdot f \cdot L \cdot V^2}{2 \cdot g \cdot D} , \qquad m = \frac{\pi/4 \cdot D^2}{\pi \cdot D} = D/4 .$$

Chezy equation

$$h_f = \frac{f'}{w} \times \frac{P}{A} \times L \times V^2 .$$

A - Area of cross section of pipe.

P - wetted perimeter of pipe.

Here Hydraulic mean depth $m = A/P$.

$$m = \frac{A}{P} = \frac{\frac{\pi}{4} \cdot d^2}{\pi d} = \frac{d}{4}.$$

So $\frac{A}{P} = \frac{d}{4}$ $\qquad A/P = m$ $\qquad P/A = 1/m.$

$$h_f = \frac{f'}{\rho . g} \times \frac{1}{m} \times L . V^2 .$$

$$V^2 = h_f . \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$V^2 = \frac{\rho . g}{f'} \times m \times \frac{h_f}{L} .$$

$$V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} .$$

$$V = \sqrt{\frac{\rho . g}{f'}} . \sqrt{m . \frac{h_f}{L}}$$

$\sqrt{\frac{\rho g}{f'}} = C$ Chezy constant.

$$V = C \times \sqrt{m \times i}$$

$\frac{h_f}{L} = i$

Loss of head / unit length of pipe

$h_f / L = i = C.$

Friction factor:

Expression for Co-efficient of friction in Terms of shear stress.

$$P_1 A - P_2 A - F_1 = 0$$

$$(P_1 - P_2) A = F.$$

$$\frac{\pi . d^2}{4} . (P_1 - P_2) = \tau_0 \times \pi d \times L$$

Cancelling πd from the both side

$$(P_1 - P_2) . d/_4 = \tau_0 \times L.$$

$$P_1 - P_2 = \frac{4 \tau_0 \times L}{d} \qquad \frac{P_1 - P_2}{\rho . g} = 4 . \frac{f . L . v^2}{2 . g . D}$$

$$\frac{2 \tau f'}{\rho . g} \Rightarrow \frac{2 . \tau . f/_2}{g} = f \qquad \boxed{f = \frac{2 \tau}{\rho . v^2}}$$

So $f = \frac{2 \tau f'}{\omega}$ = friction faction

Moody's diagram:

both are 4 marks

It is plotted between various values of friction factor (f), Reynolds number (Re) and relative roughness (k/D).

In turbulant flow, we can find the friction factor from the moody's diagram.

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(D/_k \right).$$

$k/_D$ = Roughness value for the diff. material.

$$\frac{0.035}{0.015} \quad \text{min.}$$

Major losses - flowing condition some fluid energy will lost.

I. major Energy loss:

i) Loss of energy due to friction - Darcy weisbach formula.

$$h_f = \frac{4 \cdot f \cdot L \cdot v^2}{2 \cdot g \cdot D}.$$

h_f - loss of head due to friction

f - co-efficient of friction

L - length of pipe

v - mean velocity of flow

D - Diameter of pipe.

II. M. Loss of head due to friction in pipe - Chezys formula.

$$h_f = \frac{f'}{w} \times \frac{P}{A} \times L \times v^2$$

A - Area of cross section of pipe

P - wetted perimeter of pipe

L - length of pipe.

v - velocity of fluid flow.

f' - co-efficient of friction

w - sp. weight of fluid.

D - diameter of pipe

h_f - Loss of head due to friction.

Minor losses (b) PORK.

I. Loss of head due to sudden Enlargement (h_e)

Consider a liquid flowing through a pipe which has sudden enlargement

Here

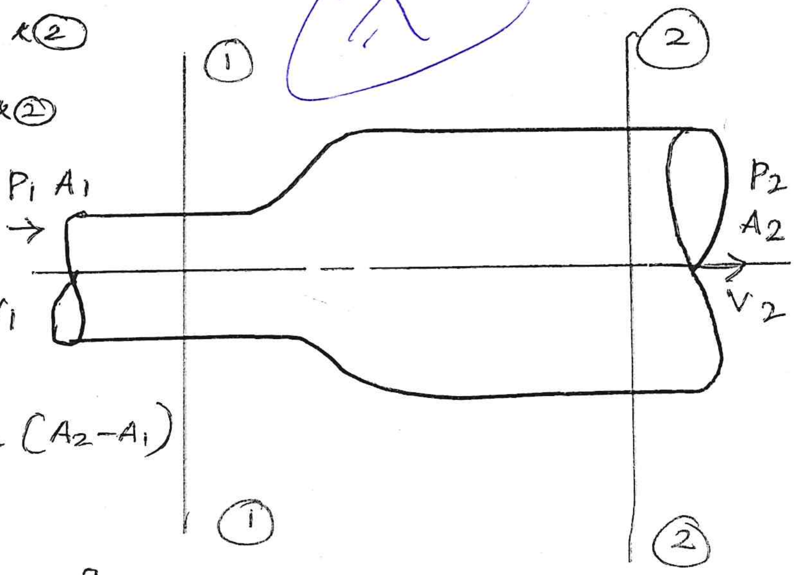
P_1 & P_2 – pressure at section ① & ②

V_1 & V_2 – velocity at section ① & ②

A_1 & A_2 – Area at section ① & ②

h_e – Loss of head due to sudden enlargement

P^1 – pressure of liquid area $(A_2 - A_1)$



Use – Ber. eqn.

$$\frac{P_1}{\rho . g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho . g} + \frac{V_2^2}{2g} + z_2 + h_e \; ; \left[\text{Here } z_1 = z_2\right]$$

$$\frac{P_1}{\rho . g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho . g} + \frac{V_2^2}{2g} + h_e$$

$$\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] + \left[\frac{P_1}{\rho . g} - \frac{P_2}{\rho . g}\right] = h_e \quad \text{——— (1)}$$

$$F_x = P_1 . A_1 + P^1 (A_2 - A_1) - P_2 A_2 .$$

Here, $P^1 = P_1$

$$F_x = P_1 A_1 + P_1 (A_2 - A_1) - P_2 . A_2 .$$

$$= \cancel{P_1 A_1} + P_1 . A_2 - \cancel{P_1 A_1} - P_2 . A_2$$

$$= (P_1 - P_2) . A_2 \quad \text{——— (2)}$$

N-S

Momentum of liquid / sec at section 1-1.

= mass × velocity

$= \rho A_1 V_1 \times V_1$

$= \rho . A_1 . V_1^2$

Momentum of liquid / sec at section 2 $\rho . A_2 . V_2^2$

Change of momentum $= \rho . A_2 . V_2^2 - \rho A_1 . V_1^2$

use cont. equation $A_1 V_1 = A_2 V_2$

$$A_1 = \frac{V_2 A_2}{V_1}$$

$$= \rho . A_2 V_2^2 - \rho . A_1 V_1^2$$

$$= \rho . A_2 . V_2^2 - \rho . \frac{A_2 V_2}{\cancel{V_1}} . V_1^{\cancel{2}}$$

$$= \rho . A_2 . V_2^2 - \rho . A_2 . V_2 . V_1$$

$$= \rho . A_2 (V_2^2 - V_1 . V_2) \quad \text{—— (3)}$$

Equate (2) & (3)

$$(P_1 - P_2) \cancel{A_2} = \rho . \cancel{A_2} . (V_2^2 - V_1 . V_2)$$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 . V_2$$

÷ by g $\quad \frac{P_1 - P_2}{\rho . g} = \frac{V_2^2 - V_1 V_2}{g}$

Sub. $\frac{P_1 - P_2}{\rho g}$ value in equation (1)

$$\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] + \frac{V_2^2 - V_1 V_2}{g} = h_e$$

$$\frac{V_1^2 - V_2^2 + 2V_2^2 - 2V_1 V_2}{2g} = \frac{V_1^2 + V_2^2 - 2 V_1 . V_2}{2g}.$$

$$h_e = (V_1 - V_2)^2 / 2g$$

II. <u>Loss of head</u> <u>due to</u> <u>sudden</u> <u>contraction</u>

$$h_c = \frac{V_2^2}{2g} \left[\frac{1}{C_c} - 1\right]^2$$

Co-efficient of Contraction $\left[\frac{1}{C} - 1\right]^2 = k.$

$$h_c = k . \frac{V_2^2}{2g}$$

Take $C_c = 0.62$.

$$h_c = 0.375 . \frac{V_2^2}{2g}$$

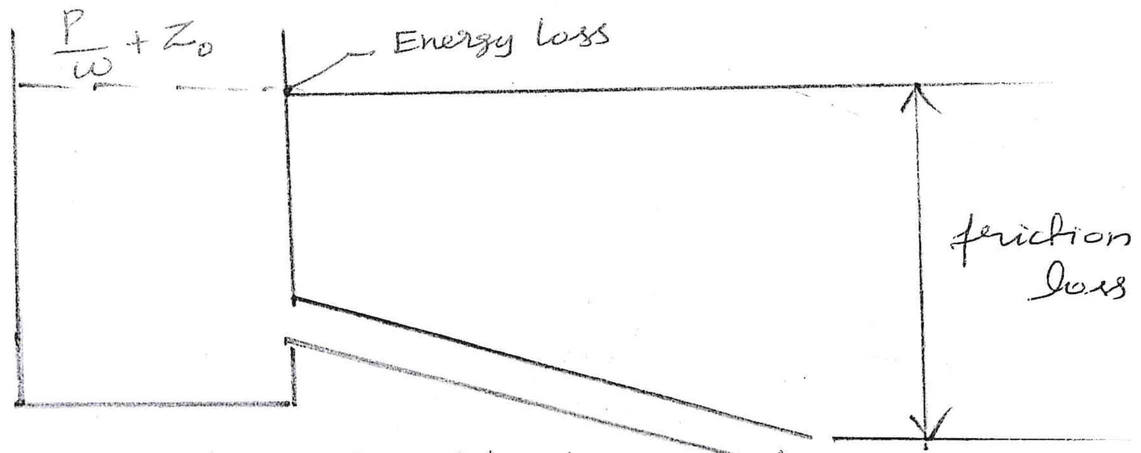
$k = 0.375.$

Hydraulic & Energy gradient lines

Hydraulic Gradient Line: (H.G.L)

A line which is the sum of $\left(\frac{P}{w}+z\right)$ with reference to the datum line is known as hydraulic gradient line.

$$H.G.L = \left[\frac{P_1}{w} + z_1 + \frac{V_1^2}{2g}\right] - \frac{V_2^2}{2g}.$$

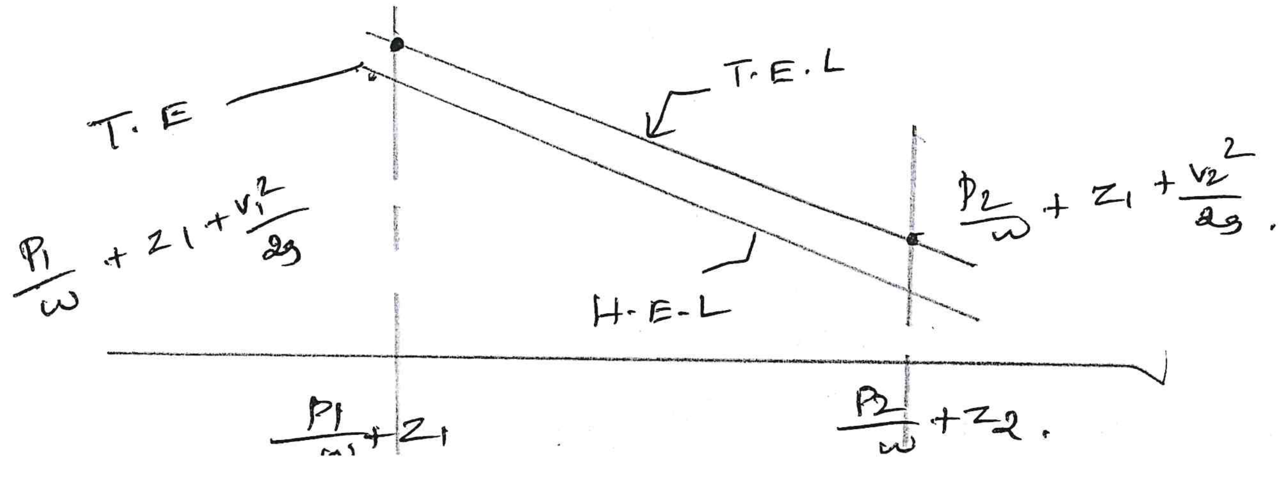


Total Energy head at inlet $(h_i + h_e + h_f)$

$$\left(\frac{P}{w} + z + \frac{V_1^2}{2g}\right) - (h_i + h_e + h_f)$$

Total Energy gradient Line (T.E.L)

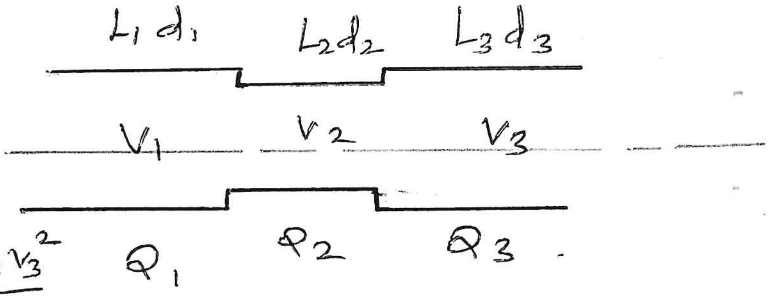
The different sections of the pipes the total energy is plotted to scale and joined by a line, the line is defined as called T.E.L.



Pipes in series

(2)

For diff. dia of pipes



$$Q = Q_1 = Q_2 = Q_2$$

Total loss of head

$$H = \frac{4f_1 L_1 V_1^2}{2gD_1} + \frac{4f_2 L_2 V_2^2}{2gD_2} + \frac{4f_3 L_3 V_3^2}{2gD_2}$$

$$f_1 = f_2 = f_3.$$

$$H = \frac{4f}{2g}\left[\frac{L_1 V_1^2}{D_1} + \frac{L_2 V_2^2}{D_2} + \frac{L_3 V_3^2}{D_3}\right]$$

So for pipe in series.

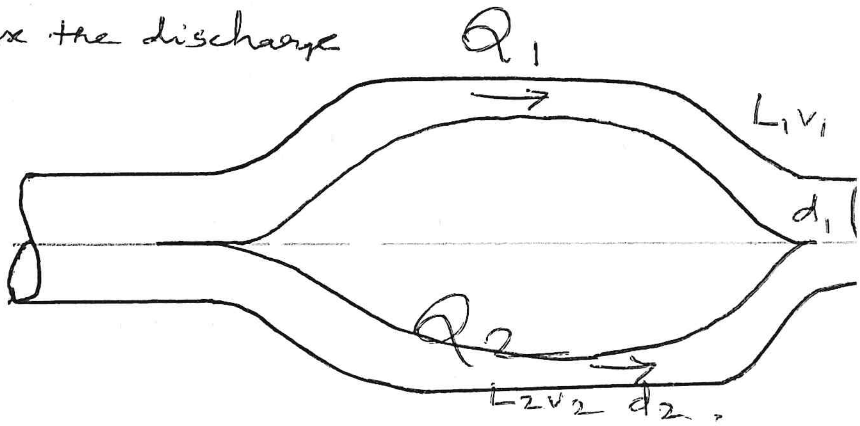
$$Q = Q_1 = Q_2$$

$$h_f = h_{f_1} + h_{f_2}$$

Pipes in parallel

(2)

To increase the discharge



Rate of flow

$$Q = Q_1 + Q_2.$$

Here $h_f = h_{f_1} = h_{f_2}$.

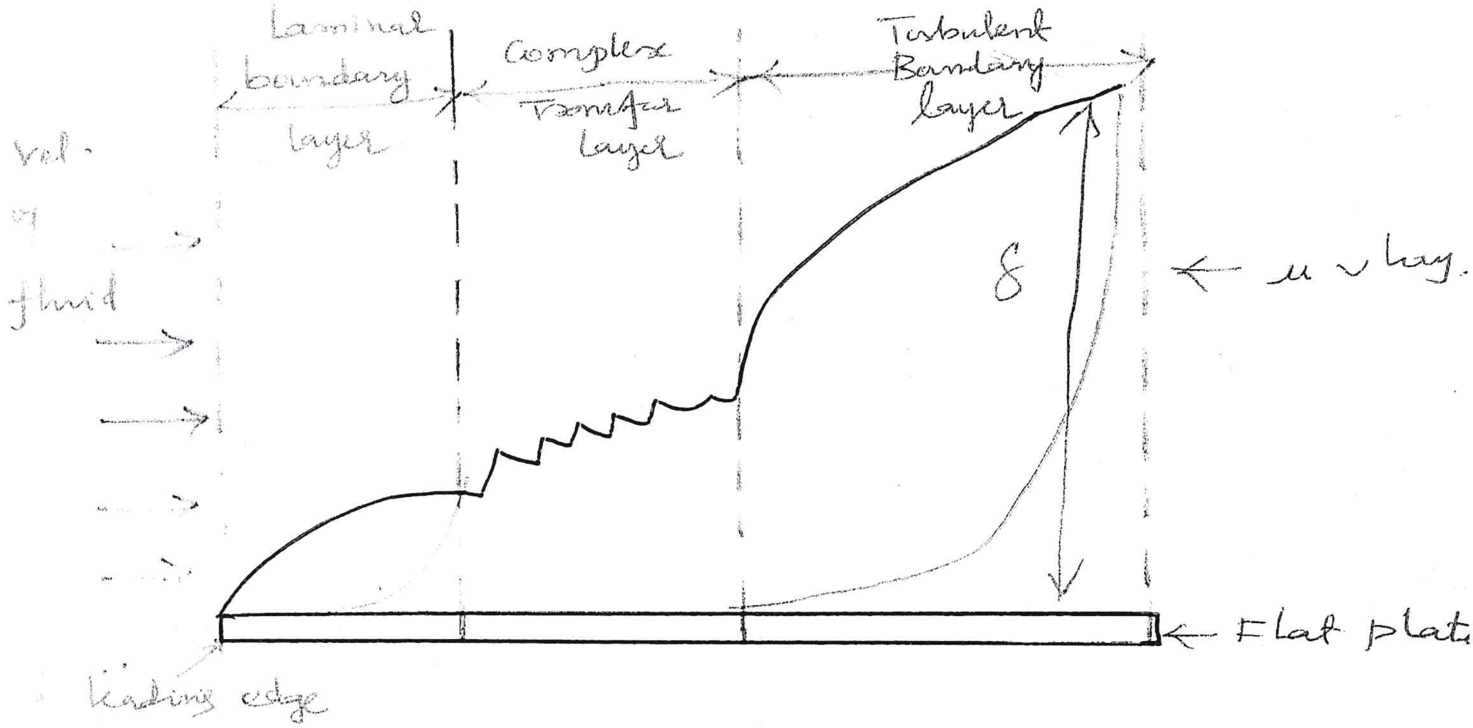
$$h_f = 4\frac{f_1 L_1 V_1^2}{2 \cdot g \cdot D_1} = 4\frac{f_2 \cdot L_2 V_2^2}{2 \cdot g \cdot D_2^2} \qquad f_1 = f_2.$$

$$\frac{L_1 V_1^2}{D_1} = \frac{L_2 V_2^2}{D_2} \quad \text{eqn.}$$

Boundary layer   Theory   Som..   2

The velocity of moving fluid distance will vary from zero at the surface of solid boundary to the velocity V (max). This variable flow of fluid layer is Boundary layer.

Consider the boundary layer formed on a flat plate kept to flow of fluid and velocity V.



δ - thickness of boundary layer

characteristics

- when δ increases as distance from leading edge x increases, ν increase
- δ decrease as V will increases.
- Boundary layer in Laminar

$$Re = \frac{Ux}{\nu} < 5 \times 10^5 \qquad Re = \rho \cdot \frac{Ux}{\mu}$$

$$\tau = \mu\left(\frac{U}{\delta}\right)$$

- Boundary layer in Turbulent

$$\frac{Vx}{\nu} > 5 \times 10^5 \qquad \delta_{lam} = \frac{5x}{\sqrt{Re}}$$

$$\delta_{turb} = \frac{0.377x}{(R)^{1/5}}$$

<u>Boundary layer thickness</u>: (δ)

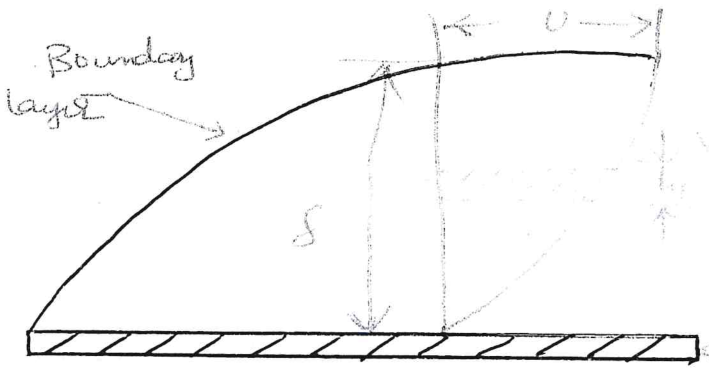
The velocity within the boundary layer increases from zero at the boundary surface and the velocity of the main stream is constant level.

So the thickness of the boundary layer is defined as that distance from the boundary in which reaches 99% of the velocity of the free stream

<u>I. Displacement thickness</u> (δ^*)

Let fluid of density ρ.

flow past a plate with velocity U



U = max. velocity

dy - elementary strip thickness

δ - displacement thickness

Take unit width, the max flow/sec through the elementary strip (with plate)

$$m = \rho . u . dy$$

without plate - Here plate is not there

$$m = \rho . U . dy$$

Reduction mass flow through the strip

$$\rho\,(U-u)\cdot dy \quad \text{——} \; (2)$$

Here,

$U-u$ = defect of velocity.

for whole plate, mass reduction.

$$\int_0^{\delta} \rho\,(U-u)\,dy \quad \text{——} \; (3)$$

Condition $\delta^* = U$

Loss of mass of fluid / sec.

$$\rho \cdot U \cdot \delta^* \quad \text{——} \; (4)$$

Equate (3) & (4)

$$\rho \cdot U \cdot \delta^* = \int_0^{\delta} \rho\,(U-u)\cdot dy$$

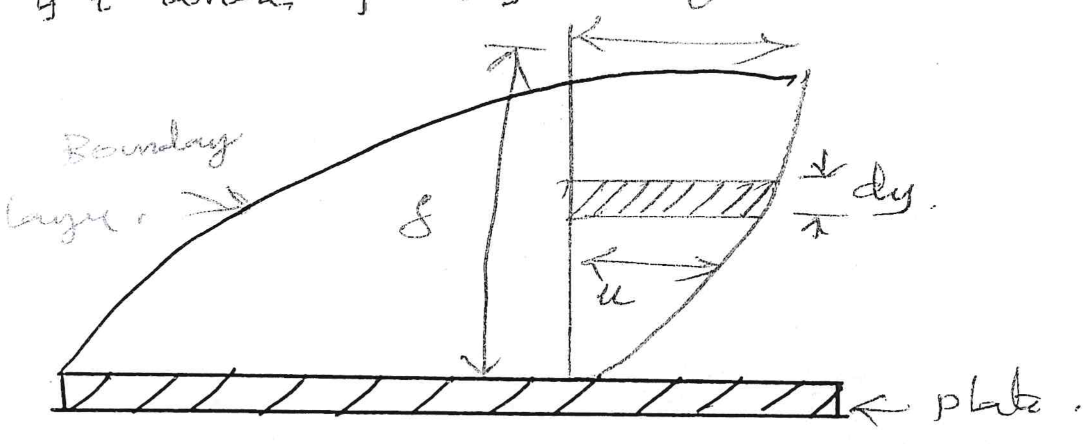
$$\delta^* = \int_0^{\delta} \frac{\rho \cdot (U-u)}{\rho \cdot U}\cdot dy$$

$$\delta^* = \int_0^{\delta} \left(1-\frac{u}{U}\right)\cdot dy.$$

Note: δ^* = distance from surface of Boundary layer.

II. Momentum thickness (θ)

It is defined as the distance through which the total loss of momentum / second be equal to if it where passing a plate.



mass flow / sec through the elementary strip

$$\rho \cdot u \cdot dy .$$

Take, momentum / sec of this fluid.
(inside the B-layer)

$$= \rho \cdot u \cdot dy \times u$$

$$= \rho \cdot u^2 \cdot dy$$

Momentum / sec → (Before entry the B. layer)

$$= \rho \cdot u \cdot U \cdot dy$$

So, Loss of momentum / sec,

$$= \rho u \cdot U \cdot dy - \rho \cdot u^2 \cdot dy .$$

$$= \rho \cdot u \, (U - u) \cdot dy .$$

Total loss of momentum / sec.

$$\int_0^{\delta} \rho \cdot u \, (U - u) \cdot dy \quad \text{——} \ (1)$$

Here,

θ = distance of fluid flow at velocity V.

The loss of momentum / sec with θ,

$$\rho \cdot \theta \cdot U^2 \quad \text{——} \ (2)$$

Equate (1) & (2)

$$\rho \cdot \theta \cdot U^2 = \int_0^{\delta} \rho \cdot u \cdot (U - u) \cdot dy$$

$$\theta = \int_0^{\delta} \frac{\cancel{\rho} \cdot u \cdot (U - u)}{\cancel{\rho} \cdot U^2} \cdot dy$$

$$\theta = \int_0^{\delta} \frac{u}{U} \left[1 - \frac{u}{U} \right] dy .$$

Displacement thickness (δ^*)

It is the distance, measured perpendicular to the boundary by which the free stream is displaced on accant of formulation of boundary layer.

Problem ①:

A plate of length 750 mm and width 250 mm has been placed longitidinally in a stream of crude oil which flows with a velocity of 5 m/sec. If the oil has a specific gravity of 0.8 and kinematic viscosity of 1 stroke. Calculate

1) Boundary layer thickness at the middle of plate
2) Shear stress at the middle of plate
3) Friction drag on one side of the plate.

Given:

Plate 750 mm x 250 mm = 0.75 x 0.25 m

oil sg = 0.8

$U = V = 5 \, m/sec.$

$\nu = 1$ Stroke $= 1 \times 10^{-4} \, m^2/sec.$

Solution

$$\frac{VL}{\nu} = Re = \frac{VD}{\nu} = \frac{5 \times 0.75}{1 \times 10^{-4}} = 0.37 \times 10^5$$

$$Re = 5 \times 10^5 = \frac{Ux}{\nu}$$

i) $\delta = \dfrac{5x}{\sqrt{Re}}$ $\qquad x = \dfrac{0.75}{2} = 0.375\ m$

$= \dfrac{5 \times 0.375}{\sqrt{0.37 \times 10^5}} = 2.65 \times 10^{-3}\ m.$

ii) Shear stress (τ_o)

$\tau_o = \mu \left(\dfrac{U}{\delta}\right)$ $\qquad \dfrac{\mu}{\rho} = \nu$

$= 0.075 \left(\dfrac{5}{9.68 \times 10^{-3}}\right)$

$\tau_o = 38.73\ N/m^2.$

iii) Friction drag on one side of the plate

Force = Stress × area

$= 141.5 \times (0.75 \times 0.25)$

$F. = 7.2\ N$

Problem 2

The velocity distribution in the boundary layer is given by $\dfrac{u}{U} = \dfrac{y}{\delta}$, where u = velocity at a distance y from the plate and $u = U$ at $y = \delta$.

Determine.

i) displacement thickness

ii) momentum thickness

iii) Energy thickness

iv) $\dfrac{\delta^*}{\theta}$

Solution:

i) Displacement thickness δ^*

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) \cdot dy \qquad \frac{u}{U} = \frac{y}{\delta}$$

$$= \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) \cdot dy$$

$$= \left[y - \frac{y^2}{2 \cdot \delta} \right]_0^{\delta}$$

$$= \left[\delta - \frac{\delta^2}{2\delta} \right] = \delta - \delta/2 .$$

$$\delta^* = \delta/2$$

ii) Momentum thickness (θ)

$$\theta = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) \cdot dy$$

$$= \int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y}{\delta}\right) \cdot dy .$$

$$= \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) \cdot dy \Rightarrow \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} \right]_0^{\delta} .$$

$$= \left[\frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} \right]$$

$$\theta = \delta/2 - \delta/3 \Rightarrow \delta/6$$

iii) Energy thickness (δ_e)

$$\delta_e = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) \cdot dy$$

$$= \int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y^2}{\delta^2}\right) \cdot dy = \int_0^{\delta} \left[\frac{y}{\delta} - \frac{y^3}{\delta^3} \right] \cdot dy$$

$$= \left(\frac{y}{\delta} - \frac{y^3}{\delta^3} \right)_0^\delta = \left[\frac{y^2}{2 \cdot \delta} - \frac{y^4}{4 \cdot \delta^3} \right]_0^\delta$$

$$= \frac{\delta^2}{2\delta} - \frac{\delta^4}{4 \cdot \delta^3}$$

$$= \frac{1}{4} \cdot \delta$$

iv) $$\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6}$$

$$= 3$$ Ans.

Fluid dynamics:

It is study of fluid motion involves the considesation of both the kinematrics (without force) and kinetrics (with force).

Laminar flow through pipes

Problem 1 Water at 0.001 N-s/m² is flowing through a 300 m long G.I pipe of inner diameter 75 mm. Calculate the drop in pressure head for the flow rate of 8 Lps, using Moody diagram.

Solution:

$\mu = 0.001 \ N\text{-}s/m^2$.

$Q = 8 \ Lps = 8 \times 10^{-3} \ m^3/sec$.

$L = 300m$

$D = 75mm = 0.75m$

$$V = Q/A = \frac{8 \times 10^{-3}}{\pi/4 \times 0.075^2} = 1.8 \text{ m/sec}.$$

kinematic viscosity $\nu = \frac{\mu}{\rho} = \frac{0.001}{1000} = 1 \times 10^{-6}$

$$Re = \frac{V D}{\nu} = \frac{1.8 \times 0.075}{1 \times 10^{-6}}$$

$$Re = 1.35 \times 10^5$$

$1.35 \times 10^5 > 2000$, so the flow is Turbulent

Roughness $k/D = \frac{0.015}{75 \times 10^{-3}} = 2 \times 10^{-4}$

from moody diagram $f = 0.0047$

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{2 \cdot g \cdot D} = \frac{4 \times 0.0047 \times 300 \times (1.8^2)}{2 \times 9.81 \times 0.075}$$

$h_f = 12.57$ m.

Turbulent flow

Here, $Re > 4000$

$$Re = \frac{\rho \cdot U \cdot D}{\mu}$$

In turbulent flow, the fluid motion is irregular and it will be complete mixing of fluid.

Reynold number (Re)

$$\tau = \rho \cdot u' v'$$

Problem 2: The rate of flow of water through a horizontal pipe is 0·25 m^3/sec. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine,

i) Loss of head due to sudden enlargement

ii) Pr. intensity in the large pipe

iii) Power lost due to enlargement

Solution:

$$Q = 0{\cdot}25 \ m^3/sec$$

$D_1 = 200$ mm $\qquad A_1 = 0{\cdot}03141 \ m^2$

$D_2 = 400$ mm $\qquad A_2 = 0{\cdot}126 \ m^2$

$$P_1 = 11{\cdot}77 \ N/cm^2 = 11{\cdot}77 \times 10^4 \ N/m^2.$$

$$V_1 = \frac{Q}{A_1} = 7{\cdot}96 \ m/sec.$$

$$V_2 = \frac{Q}{A_2} = 1{\cdot}99 \ m/sec.$$

i) Loss of head $\quad h_e = \frac{(V_1 - V_2)^2}{2g} = 1{\cdot}816$ m.

ii) $P_2 =$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e.$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e. \qquad Z_1 = Z_2.$$

$$\frac{P_2}{\rho g} = 13{\cdot}21 \times 9810 = 12{\cdot}96 \times 10^4 \ N/m^2.$$

iii) Power loss $\quad P = \rho \times g \times Q \times h_e.$

$$= 4{,}453 \text{ watts.}$$

Unit III
Dimensional Analysis and Model Studies

Fundamental Dimensions

Dimensional analysis is used to find the solution of difficult fluid problem in flow.

I. Fundamental Quantities

	Dim. symbol	
a) Mass –	M	– M
b) Length –	L	– L
c) Time –	T	– T

II. Geometric quantities

	Sym	dimensions.
a) Area –	A	– L^2
b) volume –	V –	L^3
c) moment of inertia –	I	– L^4

III. kinematic quantities (flow)

a) velocity – v – m/sec – $\frac{L}{T}$ – LT^{-1}

b) Angular velocity – ω – rad/sec – T^{-1}

c) Acceleration – a – m/sec² – $\frac{L}{T^2}$ = $L.T^{-2}$.

d. gravity – g – m/sec² – $L.T^{-2}$.

e. discharge – Q – m³/sec – $L^3.T^{-1}$.

d. kinematic viscosity – ν – m²/sec. = $L^2.T^{-1}$

IV. Dynamic Quandities: (Fluid)

i) Force - F - $kg\text{-}m/sec^2$ - $M.L/T^2 = M.L.T^{-2}$

ii) Weight - W - $kg\text{-}m/sec^2$ - $M.L.T^{-2}$

iii) Sp. weight - $w = N/m^3 = kg\text{-}m/sec^2/m^3 = M.L^{-2}.T^{-2}$

iv) Density $= \rho = kg/m^3 = M.L^{-3}$

v) Dynamic viscosity $\mu = N\text{-}s/m^2 = kg\text{-}m/sec^2 \cdot sec \cdot m^2$

$= kg/sec/m = M.L^{-1}.T^{-1}$

vi) Work / Energy / power $= N\text{-}m = kg\text{-}m/sec^2\text{-}m = kg.m^2/sec^2$

$= M.L^2.T^{-2}$.

vii) Torque $(T) = N\text{-}m = kg\text{-}m/sec^2 \cdot m = kg.m^2/sec^2$

$= M.L^2.T^{-2}$.

Dimensional homogeneity: some time big question

Dimensional homogeneity means the dimensions of each terms in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is dimensional homogeneous equation.

The powers of fundamental dimensions (i.e. L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation.

The equations are independent of the system of units.

Let us consider the equation

$$V = \sqrt{2gH}$$

Dimension of $L \cdot H \cdot S = V = \frac{L}{T} = LT^{-1}$

Dimension of $R \cdot H \cdot S = \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L}$

$$= \sqrt{\frac{L^2}{T^2}} = \sqrt{\frac{L^2}{T^2}} = L/T$$

$$= LT^{-1}$$

$$L \cdot H \cdot S = R \cdot H \cdot S$$

$$LT^{-1} = L \cdot T^{-1}.$$

So, the equation $V = \sqrt{2gH}$ is dimensionally homogeneous.

It can be used in any system of units

Example:

$$Q = 2/3 \cdot Cd \cdot \sqrt{2g} \cdot LH^{3/2}.$$

$$Q = L^3 \cdot T^{-1}$$

$$L^3 \cdot T^{-1} = \sqrt{L \cdot T^{-2}} \cdot L \cdot L^{3/2}.$$

$$= L^{1/2} \cdot T^{-1} \cdot L \cdot L^{3/2}.$$

$$= L^{(1/2 + 1 + 3/2)} \cdot T^{-1}$$

$$L^3 \cdot T^{-1} = L^3 \cdot T^{-1}$$

The left dimensions are equal to the right hand side dimensions. It can be used in any system units.

Buckingham Pi Theorem

It is used for dimensional analysis

If there are 'n' variables in a problem and these variables contain 'm' primary dimensions (M, L, T) the equation relating all the variables will have (n-m) dimensions groups.

Buckingham referred to the groups as π groups.

The final equation is

$$\pi_1 = f(\pi_2, \pi_3, \ldots \pi_{n-m}).$$

The π groups must be independent of each other and no one group should be formed by multiplying together power of other groups

Conditions:

i) Fundamental dimensions must have at lea[st] one 'm' variable

ii) Number of dimensionless group is $n-m$.

Example:

check the dimensional for the equation

$$v = u + ft.$$

$$LT^{-1} = LT^{-1} + LT^{-2}.T$$

$$= LT^{-1} + LT^{-1}$$

$$LT^{-1} = 2LT^{-1}$$

Rayleigh's method

In this method, the expression is determined for a variable depending upon maximum three or four variables only.

consider the steps.

I. The functional equation is written as.

$$X = f(X_1, X_2, X_3 \cdots X_n)$$

II. When the constant with exponents power values,

$$x = q(x_1^a, x_2^b, x_3^c \cdots x_n^z)$$

q = constant

a, b, c = power of function values.

III. Find the $a, b, c,$ values.

finally apply the value in function equation,

Note:

I. Geometric properties

length, diameter, height.

II. Flow properties.

ω, V, acc, gravity, discharge.

III. Fluid properties.

ρ, ν, Sg, μ, w

Dimensionless parameters

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension. As this is a ratio of one force to the other force, it will be a dimensionless number

① Reynold number (Laminar / Turbulent)

$$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{\rho v^2 . L^2}{\mu . L . v} = \frac{\rho v L}{\mu}.$$

② Frude number. (Used in gravity force)

$$Fr = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}} = \sqrt{\frac{\rho . L^2 . v^2}{\rho . L^3 . g}} = \sqrt{\frac{v^2}{g . L}}.$$

③ weber number (Surface tension force)

$$We = \sqrt{\frac{\text{inertia force}}{\text{Surface tension force}}} = \sqrt{\frac{\rho . L^2 . v^2}{\sigma . L}} = \sqrt{\frac{\rho . L . v^2}{\sigma}}$$

④ Euler number (Pressure formed used area).

$$Eu = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}} = \sqrt{\frac{\rho . L^2 . v^2}{L^2}} = \frac{v}{\sqrt{\rho}}$$

⑤ Mach number (Air crafted used area)

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho . L^2 . v^2}{K . L^2}}$$

$$= \sqrt{\rho . v^2 / K}$$

Model study:

For predicting the performance of the hydraulic structures (such as dams, spillways etc) or hydraulic machines (turbine, pumps), before actually constructing or manufacturing, models of the structures or machines are made and tests are performed on them to obtain the desired information.

It is used to find performance of structures.

Similitude

It is defined as the complete similarity between model and prototype.

Types of similarities:

1. Geometric similarity (shape) $= \frac{\text{prototype}}{\text{model}}$; $L_r = \frac{L_p}{L_m}$
2. Kinematic similarity (Time) $= T_r = \frac{T_p}{T_m}$
3. Dynamic similarity (force) $= F_r = \frac{(F_i)_p}{(F_i)_m} = \frac{(\text{Int. force})_m}{(\text{Int. force})_p}$.

Similarity laws:

It said that the dynamic similarity is also known as model law (or) similarities law.

Types of law

1) Reynolds model law
2) Froude model law
3) Euler model law
4) Weber model law
5) Mach model law

Distorted and Undistorted Models.

The hydraulic models are classified as

1. Undistorted models
2. Distorted models

Undistorted Models

Undistorted models are those models which are geometrically similar to their prototypes or in other words if the scale ratio for the linear dimensions of the model and its prototype is same the model is called Undistorted model.

The behaviour of the prototype can be easily predicted from the results of undistorted model.

Distorted models:

i) A model is said to be distorted if it is not geometrically similar to its prototype.

ii) For a distorted model different scale ratio for the linear dimensions are adopted.

For example, in case of rivers, harbours, reservoirs etc, two different scale ratios,

* one for the horizontal dimensions
* other for vertical dimensions

iii) Thus the models of rivers, harbours and reservoirs will become as distorted models.

iv) If for the river, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted.

Then the depth of water in the model of the river will be very-very small which may not be measured accurately.

Advantages of Distorted models

1. The vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained
4. Give useful information for model type.

Scale Ratios for Distorted Models theory.

As mentioned above, two different scale ratios, one for horizontal dimensions and another one for vertical dimensions.

$(L_r)_H$ = Scale ratio for horizontal dimension of $\frac{\text{Prototype}}{\text{model}}$

$$(L_r)_H = \frac{L_p}{L_m} = \frac{B_p}{B_m} \quad \Big\} \text{ Horizontal dimensions}$$

$(L_r)_V$ = scale ratio for vertical dimension

$$(L_r)_V = \frac{h_p}{h_m} = \frac{\text{Prototype}}{\text{model}}$$

The scale ratio for velocity, area, discharge are $(L_r)_H$ and $(L_r)_V$.

1. <u>Scale ratio for velocity</u>

V_p = velocity in prototype
V_m - velocity in model

$$\frac{V_p}{V_m} = \frac{\sqrt{2gh_p}}{\sqrt{2gh_m}} = \sqrt{\frac{h_p}{h_m}} = \sqrt{(L_r)_V}$$

2. <u>Scale ratio for area of flow</u>

A_p = Area of flow in prototype $= B_p \times h_p$
A_m = Area of flow in model $= B_m \times h_m$

$$\frac{A_p}{A_m} = \frac{B_p \times h_p}{B_m \times h_m} = \frac{B_p}{B_m} \times \frac{h_p}{h_m}$$
$$= (L_r)_H \times (L_r)_V .$$

3. <u>Scale ratio for Discharge</u>

Q_p = Discharge through prototype $= A_p \times V_p$
Q_m = Discharge through model $= A_m \times V_m$.

$$\frac{Q_p}{Q_m} = \frac{A_p \times V_p}{A_m \times V_m} = (L_r)_H \times (L_r)_V \times \sqrt{(L_r)_V}$$
$$= (L_r)_H \times \left[(L_r)_V\right]^{3/2} .$$

problem ①

The discharge through a weir is $1.5\ m^3/sec$. Find the discharge through the model of the weir if the horizontal dimension of the model $= 1/50$ the horizontal dimension of the prototype and vertical dimension of the model $= 1/10$ the vertical dimension of the prototype

Given :

Discharge through weir (prototype) $Q_p = 1.5\ m^3/s$

Horizontal dimension of model $= \frac{1}{50} \times$ Horizontal dim. of prototype

Solution :

$$\frac{\text{Horizontal dim. of Prototype}}{\text{Horizontal dim. of model}} = 50 \text{ (or) } (L_r)_H = 50$$

Vertical dimensions of model $= \frac{1}{10} \times$ Vertical dimension of prototype

$$\frac{\text{vertical dim. of Prototype}}{\text{Vertical dim. of model}} = 10$$

$$(L_r)_V = 10$$

using eqn, we get $\frac{Q_p}{Q_m} = (L_r)_H \times \left[(L_r)_V\right]^{3/2}$

$$= 50 \times 10^{3/2} = 1581.14.$$

$$Q_m = \frac{Q_p}{1581.14} = \frac{1.50}{1581.14} = 0.000948\ m^3/sec.$$

problem (2)

The pressure difference Δp in a pipe of diameter D and length L due to turbulent flow depends on the velocity V, viscosity μ, density ρ and Roughness k. Using Buckingham π-Theorem find the expression for Δp.

Solution :

The function

$$\Delta p = f(D, L, V, \mu, \rho, k)$$

$$f_1(\Delta p, D, L, V, \mu, \rho, k) = 0 \quad \text{--- (1)}$$

so number of variables $n = 7$

Number of terms $= n - m = 7 - 3 = 4$ terms

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad \text{——} \; (2)$$

Choose D, V, ρ are repeated variables

π term $(m+1)$

$$\pi_1 = (D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot \Delta p)$$

$$\pi_2 = (D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot L)$$

$$\pi_3 = (D^{a_3}, V^{b_3}, \rho^{c_3} \cdot \mu)$$

$$\pi_4 = (D^{a_4}, V^{b_4}, \rho^{c_4}, k)$$

Find π_1 term:

$$\pi_1 = (D^{a_1} \cdot V^{b_1}, \rho^{c_1} \cdot \Delta p)$$

$$M^0 L^0 T^0 = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot [ML^{-1} \cdot T^{-2}]$$

Equate MLT,

For M $\quad 0 = c_1 + 1 \; ; \; c_1 = -1$

For L $\quad 0 = a_1 + b_1 - 3c_1 - 1$

For T $\quad 0 = -b_1 - 2 \; ; \; b_1 = -2$

so $a_1 = 0$

Substituting the value of a_1, b_1, c_1 in π_1

$$\pi_1 = (D^0, V^{-2}, \rho^{-1} \cdot \Delta p)$$

$$\pi_1 = \frac{\Delta p}{\rho \cdot V^2}$$

Π_2 Term :

$$\Pi_2 = (D^{a_2} . V^{b_2} . \rho^{c_2} . L)$$

substituting dimensions on both sides

$$M^0 L^0 T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-3})^{c_2} . L^1$$

Equate M, L, T powers on both sides.

For M $\quad 0 = c_2 \quad ; \; c_2 = 0$

For L $\quad 0 = a_2 - b_2 - 3c_2 + 1 .$

For T $\quad 0 = -b_2 \quad b_2 = 0 .$

$$a_2 = -1 .$$

Substituting the values a_2, b_2, c_2 in Π_2

$$\Pi_2 = D^{-1} . V^0 . \rho^0 . L$$

$$\Pi_2 = \left(\frac{L}{D}\right) .$$

Π_3 Term :

$$\Pi_3 = (D^{a_3} . V^{b_3} . \rho^{c_3} . \mu)$$

Apply the dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_3} . [LT^{-1}]^{b_3} . (ML^{-3}]^{c_3} . [ML^{-1} . T^{-1}]$$

Equate the power values of MLT.

For M $\quad 0 = c_3 + 1 \quad ; \; c_3 = -1$

For L $\quad 0 = a_3 + b_3 - 3c_3 - 1$

For T $\quad 0 = -b_3 - 1$

$$b_3 = -1$$

$$a_3 = -1$$

Apply the values of a_3, b_3, c_3 in π_3 term.

$$\pi_3 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\pi_3 = \frac{\mu}{D \cdot V \cdot \rho}$$

$\underline{\pi_4}$ Term :

$$\pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot k$$

Apply the dimensions on both sides.

$$M^0 L^0 T^0 = L^{a_4} \cdot (LT^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot L^1$$

Equate powers

For $M \Rightarrow 0 = c_4$; $c_4 = 0$

For $L \Rightarrow 0 = a_4 + b_4 - 3c_4 + 1$

For $T \Rightarrow 0 = -b_4$ $\quad b_4 = 0$

$$a_4 = -1$$

Apply the values of a_4, b_4, c_4 in π_4 Term

$$\pi_4 = D^{-1} \cdot V^0 \cdot \rho^0 \cdot k$$

$$\pi_4 = \frac{k}{D}$$

Sub. values of $\pi_1, \pi_2, \pi_3, \pi_4$ in eqn (2)

$$f_1 \left(\frac{\Delta p}{\rho V^2}, \frac{L}{D}, \frac{\mu}{D V \rho}, \frac{k}{D} \right) = 0$$

$$\frac{\Delta p}{\rho \cdot V^2} = \varphi \left(\frac{L}{D}, \frac{\mu}{D V \rho}, \frac{k}{D} \right)$$

$$\frac{\Delta p}{\rho \cdot V^2} = \frac{L}{D} \varphi \left(\frac{\mu}{\rho V D}, \frac{k}{D} \right).$$

$$\frac{\Delta p}{\rho} = V^2 \cdot \frac{L}{D} \varphi \left(\frac{\mu}{\rho V D}, k/D \right)$$

$\div$ by g $\quad \dfrac{\Delta p}{\rho \cdot g} = \dfrac{V^2}{g} \cdot \dfrac{L}{D} \cdot \varphi\left(\dfrac{\mu}{\rho V D}, k/D\right)$.

$$\frac{p_1 - p_2}{w} = \frac{V^2 \cdot L}{g D} \varphi\left(\frac{1}{Re}, fr\right).$$

$$\Delta p = \frac{4 f L V^2}{2 g D} \text{ Ans.}$$

Problem no (3)

The frictional torque T of a disc of diameter D rotating at a speed N in a fluid of viscosity μ and density ρ in a turbulent flow is given by $T = D^5 N^2 \rho \, \varphi\left(\dfrac{\mu}{D^2 N \rho}\right)$. Proove this by the method of dimensions.

Given:

$$T = f(D, N, \mu, \rho).$$

$$f_1(T, D, N, \mu, \rho) = 0 \quad \text{—— (1)}$$

Solution:

Dimensional Analysis

$T = M L^2 \cdot T^{-2}$

$D = L$

$N = T^{-1}$

$\mu = M \cdot L^{-1} \cdot T^{-1}$

$\rho = M L^{-3}$

Number of variables $n = 5$

No. of π terms $= n - m$

$= 5 - 3 = 2$

$$f_1(\pi_1, \pi_2) = 0 \quad \text{—— (2)}$$

$\pi_1 = (D^{a_1}, N^{b_1}, \rho^{c_2} \cdot T)$

$\pi_2 = (D^{a_2}, N^{b_2}, \rho^{c_3}, \mu)$

Π_1 Term:

$$\Pi_1 = D^{a_1} . N^{b_1}, \rho^{c_1} . T$$

$$M^0 L^0 T^0 = L^{a_1} . (T^{-1})^{b_1} . (ML^{-3})^{c_1} . (ML^2 T^{-2})$$

Equate the MLT

For $M = 0 = c_1 + 1$

$c_1 = -1$

For L $0 = a_1 - 3c_1 + 2$

For $T = 0 = -b_1 - 2$

$b_1 = -2$

$a_1 = -5$

Apply the a_1, b_1, c_1 in π_1 term

$$\Pi_1 = (D^{-5} . N^{-2} . \rho^{-1} . T)$$

$$\Pi_1 = \frac{T}{D^5 . N^2 . \rho}$$

Π_2 Term:

$$\Pi_2 = D^{a_2} . N^{b_2} . \rho^{c_2} . \mu .$$

Apply the MLT function on both sides

$$M^0 L^0 T^0 = L^{a_2} . (T^{-1})^{b_2} . (ML^{-3})^{c_2} . (ML^{-1} . T^{-1})$$

Equate powers

For $M \Rightarrow$ $0 = c_2 + 1$

$c_2 = 1$

for $L \Rightarrow$ $0 = a_2 - 3c_2 - 1$

$a_2 = -2 .$

for T $0 = -b_2 - 1$

$b_2 = -1$

Apply the a_2, b_2, c_2 values in π_2 Term.

$$\pi_2 = D^{-2} . N^{-1} . \rho^{-1} . \mu .$$

$$\pi_2 = \frac{\mu}{D^2 . N . \rho}$$

substitute the π_1, π_2 in eqn (2)

$$f_1 \left(\frac{T}{D^5 . N^2 . \rho} , \frac{\mu}{D^2 . N . \rho} \right) = 0 .$$

$$\frac{T}{D^5 . N^2 . \rho} = \phi \left(\frac{\mu}{D^2 . N . \rho} \right)$$

$$T = D^5 . N^2 . \rho \ \phi \left(\frac{\mu}{D^2 . N . \rho} \right)$$

problem 4.

A pipe diameter 1.5 m is required to transport an oil of specific gravity 0.9 and viscosity 3×10^{-2} poise at the rate 3000 lit/sec. Tests were conducted on a 15 cm diameter pipe using water 20°c. Find the velocity and rate of flow of model. viscosity of water at 20°c is 0.01 poise.

Given :

$D_p = 1.5 m$

$\mu_p = 3 \times 10^{-2}$ poise

$S_p = 0.9$

$Q_p = 3000$ lit/sec $= 0.3 \ m^3/sec$.

density of oil $\rho_p = 0.9 \times 1000$

$= 900 \ kg/m^3$.

$D_m = 15 cm = 0.15 \ m$.

$\mu_m = 1 \times 10^{-2}$ poise.

$\rho_m = 1000\ kg/m^3$.

Solution:

Reynold eqn.

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\frac{V_m}{V_p} = \frac{\rho_p}{\rho_m} \cdot \frac{D_p}{D_m} \cdot \frac{\mu_m}{\mu_p}$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}}$$

$$V_r = 3$$

$$V_p = \frac{Q_p}{A_p} = 1.697\ m/sec.$$

$$\frac{V_m}{V_p} = 3$$

$$V_m = 3 \times 1.697$$

$$= 5.091\ m/sec.$$

For model,

$$Q_m = A_m \times V_m$$

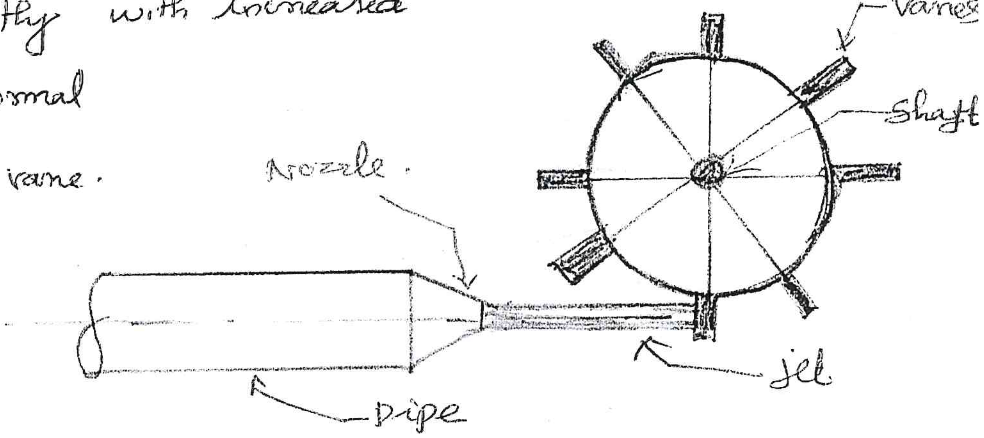
$$= 0.089\ m^3/sec.$$

Impact of Jets (2 mark)

When a jet of water flowing with a steady strikes a solid surface, the water is deflected to flow along the surface. When a jet of water is directed to hit the vanes of a particular shape, a force is exerted on the vane by the jet.

Jet Impingement upon a series of Moving flat vane

If a flat vane mounted on the periphery of a wheel, moves constantly with increased rate of speed, the normal force acting on the vane.



$$F_n = \rho A v\left[(v-u) - 0\right]$$
$$= \rho A \cdot v(v-u)$$

Workdone $W = \rho \cdot A \cdot v \cdot (v-u) \times u$

Kinetic Energy of jet $K \cdot E = \frac{1}{2} m \cdot v^2 = \frac{1}{2}(\rho \cdot A \cdot v) \cdot v^2$
$$= \frac{1}{2} \cdot \rho \cdot A \cdot v^3$$

So, Efficiency $\eta = \dfrac{\text{workdone}}{\text{Kinetic Energy}}$

$$= \frac{\rho \cdot A \cdot v \cdot (v-u)\, u}{\frac{1}{2} \cdot \rho \cdot v^3} = \frac{2u(v-u)}{v^2} \quad \text{—(1)}$$

The efficiency will be maximum when the derivate to eqn is 0.

$$\frac{d\eta}{du} = \frac{d}{du}\left(\frac{2u(v-u)}{v^2}\right) = 0$$

$$\frac{2u \times (-1) + 2(v-u) \times 1}{v^2} = 0$$

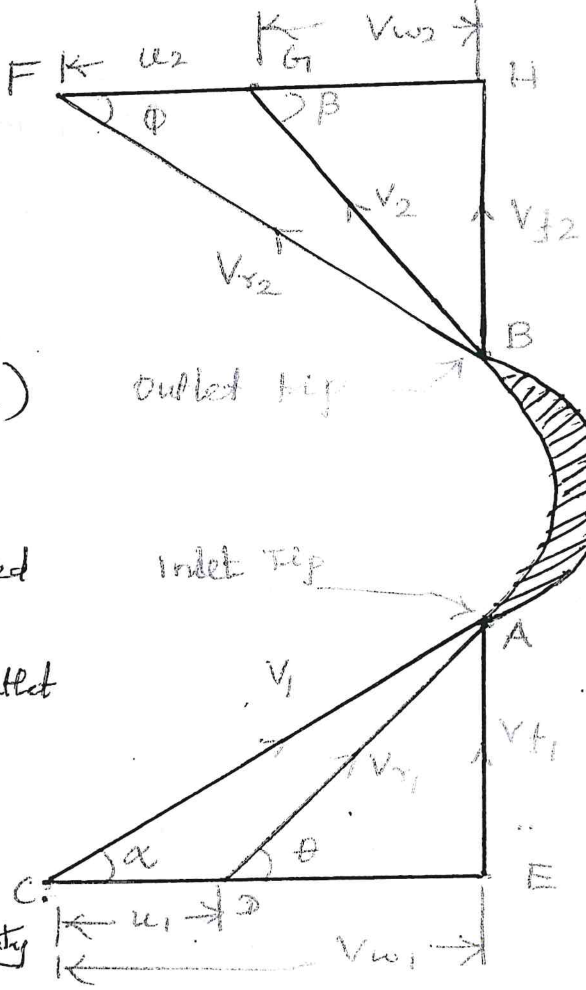
$$\frac{2}{v^2}(-u + v - u) = 0 \Rightarrow \frac{2}{v^2}(v - 2u) = 0 \Rightarrow u = \frac{v}{2}$$

Velocity Triangles

The velocity Triangles are mainly used to calculate the axial thrust Force exerted by fluid

$$F = \rho . a V_1 (V_{w_1} + V_{w_2})$$

Axial thrust $F_a = \rho . a V_1 (V_{f_1} - V_{f_2})$



Construction of velocity diagram

1. All the parameters involved in hydraulic machines are seperately listed in inlet and outlet sections.
2. From the point A, inlet
   - V_1 - Absolute velocity
   - V_{w_1} - whirl component of velocity
   - u_1 - Blade velocity
   - V_{r_1} - Relative velocity
   - V_{f_1} - velocity of flow at inlet
3. From the point A, outlet
   - V_2 - Absolute velocity
   - V_{w_2} - whirl component of velocity
   - u_2 - Blade velocity
   - V_{r_2} - Relative velocity
   - V_{f_2} - velocity of flow at outlet.
3. The angle values of both input & output velocity triangles ar

   α - jet angle at inlet

   θ - plate angle

   β - outlet angle of blade

   ϕ - Runner vane angle at outlet.

Theory of rotodynamic Machines

The rotodynamic receives the continuous motion of the fluid energy and it converts to work.

The mechanical energy converted to Hydraulic energy by the rotodynamic element.

classifications of rotodynamic machines.

1. Hydraulic Turbines
2. compressors

I. pelton wheel turbine

- Impulse turbine
- give tangent flow
- High head range, above 35m.
- Suitable for low speeds.

II. Francis Turbine

- It is a Reaction turbine
- Radial flow for suitable
- Requirement for medium head
- High speed.

III. kaplan Turbine:

- It is a axial flow turbine
- Suitable for low head
- High speed.

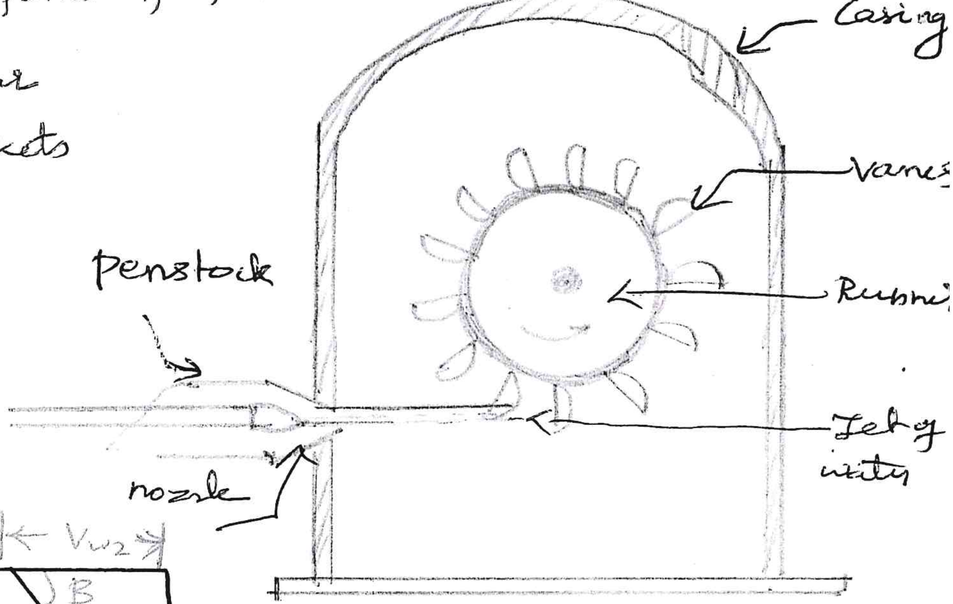
Turbine:

which one of the device convert the energy of flow water into mechanical Energy, that is called Turbine.

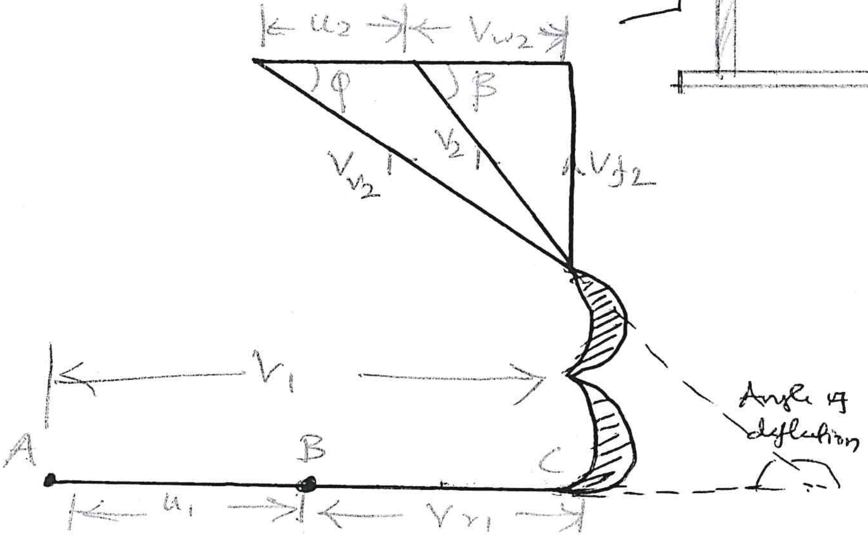
Classifications of Turbines

Pelton wheel : The pelton Turbine is a tangential flow impulse turbine. This type of turbine is used for high heads. The main parts of pelton turbine are

1. nozzle with spear
2. Runner and buckets
3. casing
4. Breaking jet



Velocity Triangle



H_g = Gross head

$h_f = \dfrac{4flv^2}{2gD}$.

D - diameter of Penstock

Here, V_1 = Velocity of jet at inlet $= \sqrt{2gH}$

$u = u_1 = u_2 = \pi DN/60$

The velocity Triangle at inlet will be a straight line where.

$V_{r1} = V_1 - u_1 = V_1 - u$.

$V_{w1} = V_1 \qquad \alpha = 0°, \theta = 0°$

From the velocity Triangle at outlet; $V_{r2} = V_1$ & $V_{w2} = V_{r2}\cos\phi - u_2$

The force exerted by jet of water

$$F_x = \rho a V_1 [V_{w1} + V_{w2}].$$

$a = \frac{\pi}{4} d^2$ area of jet

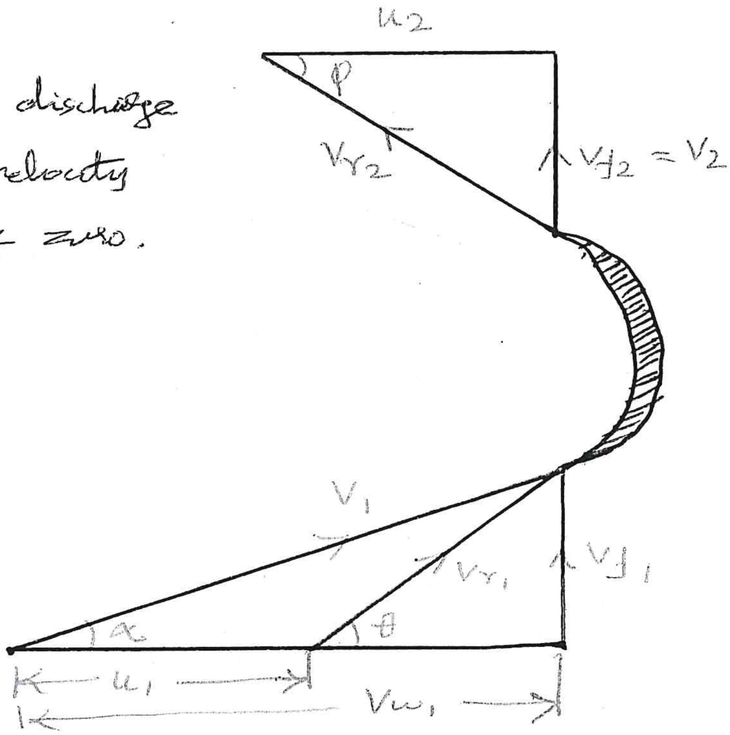
Workdone by the jet on the runner / second.

$$= F_x \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u \quad N\text{-}m/s$$

Modern Francis Turbine (mixed flow turbine)

The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine. In this modern Francis Turbine, the water enters runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the Turbine.

Velocity Triangle



In this turbine, the discharge is radial at outlet, the velocity whirl at outlet will be zero.

Hence the workdone

$$\rho Q \left[V_{w_1} u_1 \right]$$

Workdone / Second

$$= \frac{1}{g} \left[V_{w_1} u_1 \right]$$

Hydraulic efficiency

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

Important relations of Francis Turbine

1. The ratio of width of wheel to diameter

$$n = \frac{B_1}{D_1}$$

2. The flow ratio is given as.

$$\frac{V_1}{\sqrt{2gH}}$$ it varies from 0.15 to 0.30

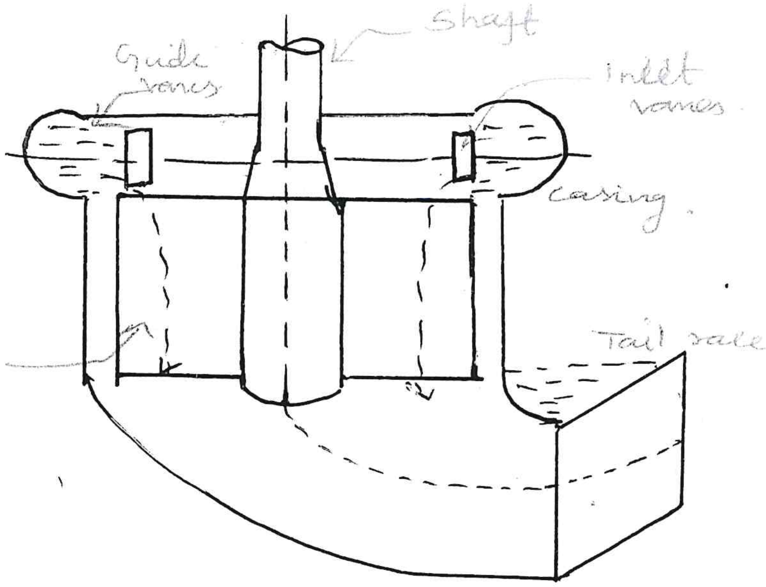
3. The speed ratio

$$\frac{u_1}{\sqrt{2gH}}$$ varies from 0.6 to 0.9.

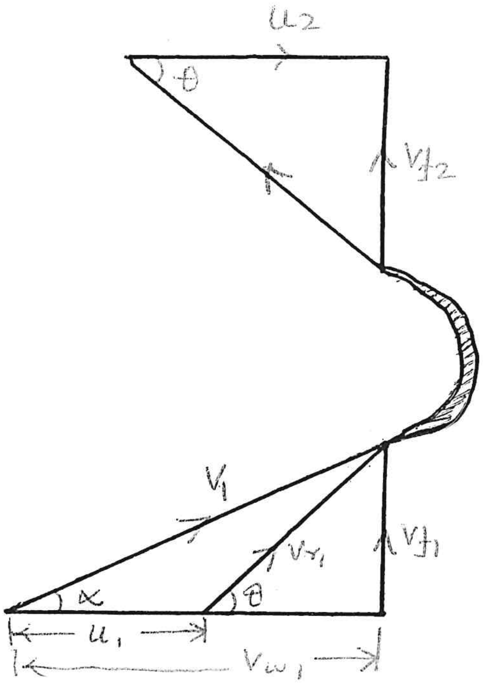
Kaplan Turbine

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. The main parts of Kaplan Turbine.

1. Scroll Casing
2. Guide vanes
3. runner of turbine (hub)
4. Draft tube



Velocity Triangle



The discharge through the runner is

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

Here,
D_o – outer diameter of runner
D_b – Diameter of hub
V_1 – velocity of flow at inlet.

Important points of Kaplan Turbine

1. The peripheral velocity at inlet and outlet are equal

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

2. The velocity of flow at inlet and outlet are equal.

$$V_{f_1} = V_{f_2}$$

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}\left(D_o^2 - D_b^2\right)$$

Workdone

Francis Turbine

The general expression for workdone follows from Euler momentum equation usual notation.

$$\text{Workdone} = \rho \cdot g \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$$

$$= \frac{w \cdot Q}{g} \left(V_{w_1} u_1 \pm V_{w_2} u_2 \right)$$

V_{w_1} & V_{w_2} - velocity of whirl at inlet & outlet.

$V_{w_2} = 0$,

$$\text{Workdone} = \frac{WQ}{g} \left(V_{w_1} u_1 \right) = \rho g V_{w_1} u_1$$

Hydraulic input of turbine $= wQH$.

Pelton wheel

Force exerted by water in the direction of motion is given by

$$F = \rho \cdot a V_1 \left(V_{w_1} + V_{w_2} \right)$$

ρ - mass density

a - area of jet $= \pi/4 \cdot d^2$

Workdone by the jet on the runner per second,

$$= F \times u$$

$$= \rho a_1 V_1 \left(V_{w_1} + V_{w_2} \right) \times u$$

weight of water striking $= \rho a_1 V_1 \times g$

workdone per second per unit weight of water striking

$$= \frac{\rho a_1 V_1 \left(V_{w_1} + V_{w_2} \right) \times u}{\rho a V_1 \times g}$$

$$= \frac{1}{g} \left(V_{w_1} + V_{w_2} \right) \times u.$$

Efficiencies

1. Hydraulic efficiency (η_h):

It is defined as the ratio of power developed by the runner to the power supplied by the water jet

$$\eta_h = \frac{\text{Power developed by the runner}}{\text{Power supplied by the water jet}}$$

$$= \frac{\rho \cdot Q (V_{w_1} \pm V_{w_2}) u}{\rho \cdot g \cdot Q \cdot H} = \frac{(V_{w_1} \pm V_{w_2}) u}{g H}$$

$$\eta_h = \frac{H_r}{H} = \frac{\text{Runner head}}{\text{Euler head}}.$$

2. Mechanical efficiency (η_{mech}):

It is the ratio of power available at the turbine shaft to the power developed by the turbine runner.

$$\eta_{mech} = \frac{\text{Power available by shaft}}{\text{Power developed by runner}} = \frac{\text{Shaft Power}}{\text{Water Power}}$$

$$\eta_{mech} = \frac{P}{w Q_a H_r}.$$

3. Volumetric efficiency (η_{vol})

It is defined as the volume of water actually striking the buckets to the total power of water by supplied by jet.

$$\eta_{vol} = \frac{\text{Quantity of water actually striking on vane}}{\text{Total water supplied by ~~vane~~ jet}}$$

$$\eta_v = \frac{Q_a}{Q}$$

4. Overall efficiency (η_o)

Overall efficiency is defined as the ratio of power available at the turbine shaft to the power available from the water jet.

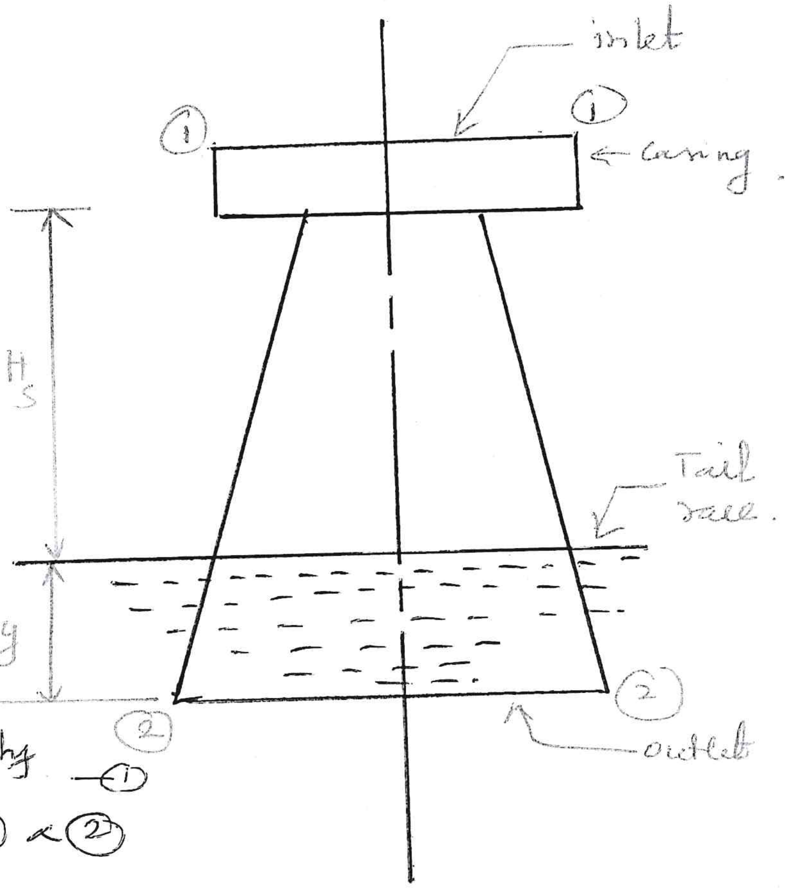
$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}} = \frac{P}{w Q H} \Rightarrow \eta_v \times \eta_h \times \eta_{mech}$$

Draft Tube

The draft tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race.

It is used for discharging water from the exit of the turbine to the tail race.

Types:
1. conical draft tube
2. simple elbow tubes
3. Moody spreading tubes.
4. Elbow draft tubes with circular inlet



Draft tube - theory

Here

H_s - vertical height of Draft tube

y - distance of bottom of tube from tail race.

Applying the Bernoulli's equation to inlet section ① and outlet section ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f \quad \text{—①}$$

h_f - loss of energy between ① & ②

$$\frac{P_2}{\rho g} = H_{atm} + y.$$

$$= \frac{P_a}{\rho g} + y$$

Substituting this value of $P_2/\rho g$ in equation —①.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + (H_s + y) = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + \cancel{y} + h_f.$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f.$$

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_f - H_s \quad \text{eqn.}$$

10 Specific Speed.

It is defined as the speed of a Turbine which is identical in shape, geometrical dimensions, blade angles, gate opeing etc.. with the actual turbine but of such a size that will develop power when working under head.

The overall efficiency of turbine.

$$\eta_0 = \frac{\text{Shaft power}}{\text{water power}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$$

where, H - head under which the turbine working

Q - Discharge through turbine

P - power developed by shaft.

$$P = \eta_0 \times \frac{\rho \times g \times Q \times H}{1000}$$

The absolute velocity, tangential velocity and head of turbine are related as

$$u \propto V, \quad V \propto \sqrt{H}$$

But Tangential velocity u is given by.

$$u = \pi D N / 60 \quad ; \quad \sqrt{H} \propto DN \text{ or } \sqrt{H}/N$$

The discharge through the turbine is

$$Q = \text{Area} \times \text{velocity}. \qquad \text{Area} = B \times D \quad \text{width} \times \text{Dia of turbine}$$

$$= D^2 \times \sqrt{H}$$

$$= \left[\sqrt{H}/N\right]^2 \times \sqrt{H}$$

$$= \frac{H}{N^2} \times \sqrt{H} = H^{3/2}/N^2$$

The specific speed of turbine, we can written as

$$N_s = \sqrt{\frac{N^2 P}{H^{5/2}}} = \frac{N\sqrt{P}}{H^{5/4}}.$$

Performance curves for Turbines

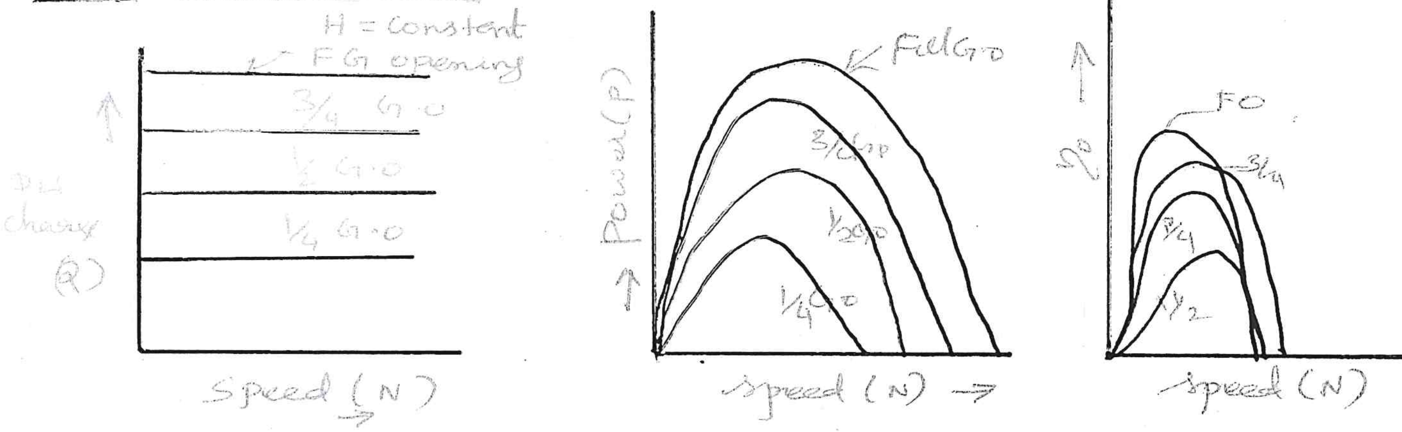
The characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions, can be known.

The important parameters which are varied during the test on a turbine are

1. Speed (N)
2. power (P)
3. Head (H)
4. Efficiency (η)
5. Discharge (Q)
6. Gate opening

The independent parameters are N, H, Q.

Main Characteristic curves:



i) The curves are obtained by maintaining a constant head and a constant gate opening (G.O) on the turbine.

ii) The speed of the turbine is varied by changing load on the turbine, for each value of the speed, power, discharge are obtained.

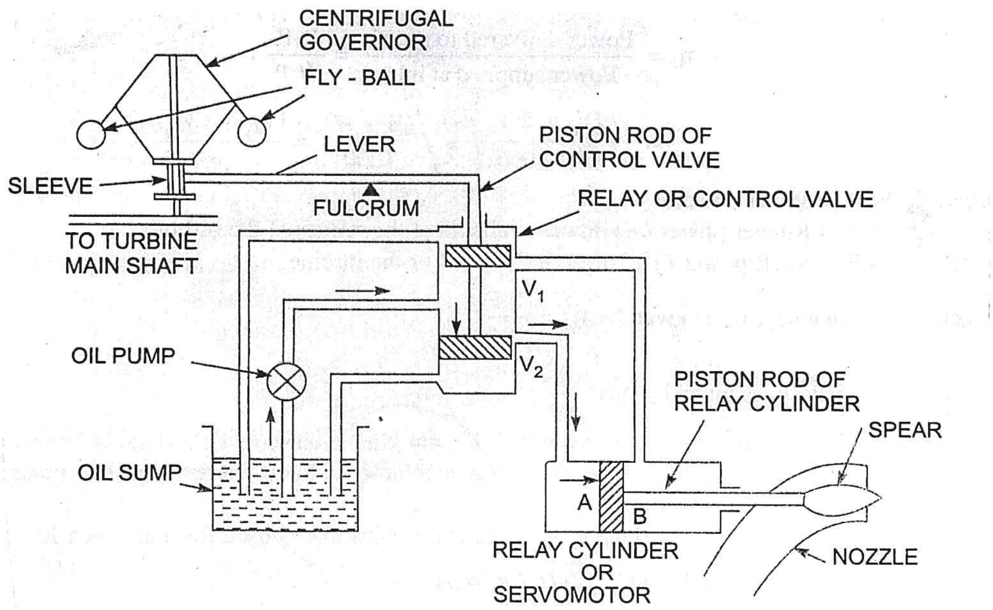
iii) The overall efficiency for each value of the speed calculated. From these readings the values of unit speed, unit power, unit discharge are determined.

iv) Out of the three independent parameters (N, H, Q) one of the parameters is kept constant (H) head and the variation of the other four parameters with respect to any one of the remaining two independent variables.

Governing of Turbines (N=C)

The governing of a turbine is defined as a operation by which the speed of the turbine is kept constant under the all conditions of working.

The Governing of pelton turbine is done by means of oil pressure governor, which consist the following parts



1. oil pump
2. centrifugal governor
3. relay or control valve
4. Relay cylinder
5. oil sump
6. spear with nozzle.

i) The oil pumped from the oil pump to control the valve under pressure flow through the valve V_1 and V_2. And we will exert force on the face A & B of the pistion of relay of the cylinder.

ii) The piston along with piston rod and spear will move towards right. This will decrease the area of flow of water at the outlet of the nozzle.

Problem ① The following data are given for a Francis turbine. Net head $H = 50\,m$, speed $N = 600\,rpm$, shaft power $= 400\,HP$, $\eta_o = 84\%$, $\eta_{hyd} = 90\%$, flow ratio $= 0.2$ breath ratio $= 0.1$,

outer diameter of runner $= 2 \times$ inner diameter

The thickness vanes occuping 5% of the circumferential area of the runner. The velocity of flow is constant at outlet and also inlet;

Determine,

i) Guide blade angle
ii) Runner vane angle
iii) Diameter of inlet & outlet of runner
iv) width of the wheel at inlet

Given

$H = 50\,m$, $N = 600\,rpm$

Shaft power $P = 400\,HP = 400 \times 735.75$
$= 294300\,W = 294.\,kW$.

$\eta_o = 84\%$ $\eta_{hyd} = 90\%$

$k_f = 0.2$, Breath ratio $n = 0.1 = \frac{B_1}{D_1} = 0.1$

$D_1 = 2 \times D_2$

$V_{f1} = V_{f2}$ $t_v = 5\% \times$ Area of runner.

Solution

$$\eta_o = \frac{P_s}{wQH}$$

$$0.84 = \frac{294}{9.81 \times Q \times 50}$$

$Q = 0.714\,m^3/sec.$ $V_{f1} = k_f \cdot \sqrt{2gH}$

$$k_f = \frac{V_{f1}}{\sqrt{2gH}} \Rightarrow 0.2 = \frac{V_{f1}}{\sqrt{2 \times 9.81 \times 50}} \Rightarrow V_{f1} = 6.264\,m/sec$$

So Actual flow area $A = 0.95 \times \pi D_1 B_1$

$$Q = A \times V$$

$$Q = 0.95 \times \pi D_1 B_1 \times V_{f1}$$

$$0.714 = 0.95 \times \pi \times D_1 \times 0.1 D_1 \times 6.264$$

$$D_1^2 = 0.382$$

$$D_1 = .618 \text{ m} \quad ; \quad D_2 = 309 \text{ mm}$$

width of wheel at inlet $B_1 = 6.2$ m.

$$u_1 = \frac{\pi D_1 N_1}{60} = 19.45 \text{ m/sec.}$$

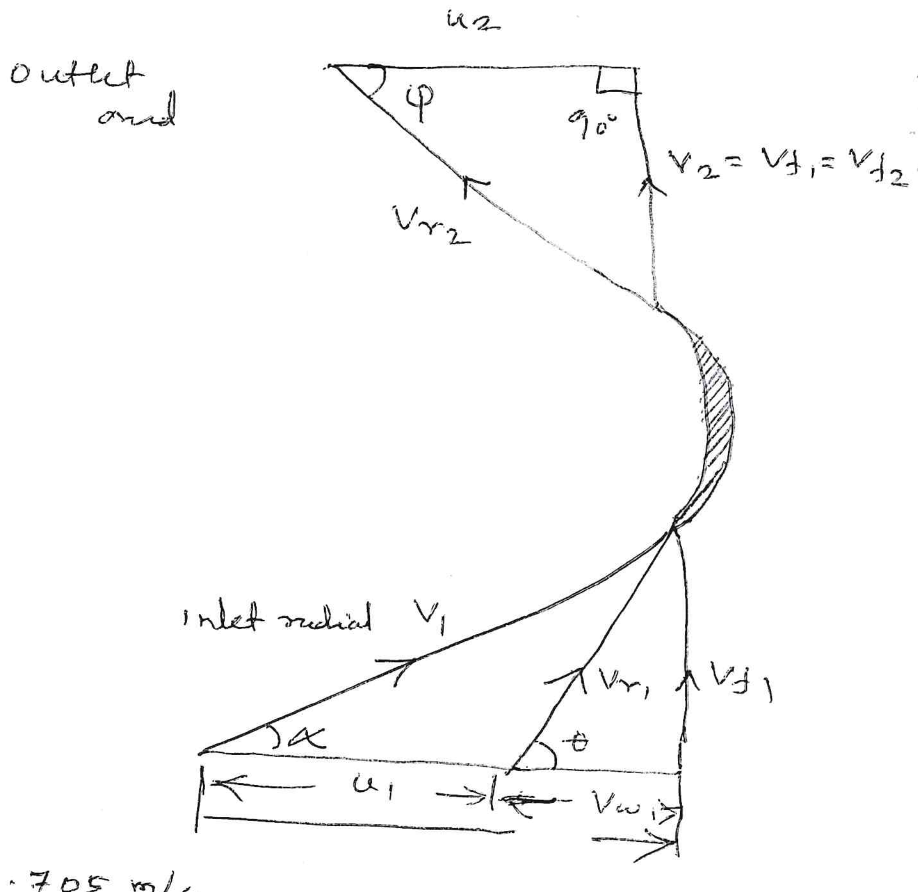
$u_1 < u_1'$

w.k.T, $\eta_h = \dfrac{V_{w1} u_1}{g \cdot H} \Rightarrow 0.90 = \dfrac{V_{w1} \times 19.45}{9.81 \times 50}$

$$V_{w1} = 22.73 \text{ m/sec.}$$

$V_{w2} = 0$

$\beta = 90^\circ$



Guide blade angle

$$\tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$\alpha = 15.4^\circ$$

Runner vane angle

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\theta = 62^\circ$$

$$u_2 = \frac{\pi D_2 N}{60} = 9.705 \text{ m/sec.}$$

Runner blades angle at outlet (φ)

$$\tan \varphi = \frac{V_{f2}}{u_2} \quad ; \quad \varphi = 32.8^\circ$$

α – Guide angle – inlet

θ = Runner angle at inlet

φ – Runner angle outlet

β – Guide angle outlet.

Problem (2) A kaplan turbine runner is to be designed to develop 7357.7 kw shaft power. The net available head is 5.5 m. Assume the speed ratio is 2.09 and flow ratio is 2.68 and the $\eta_o = 60\%$. The diameter of the boss is 1/3 rd of diameter of the runner. Find the diameter of runner and its speed with specific speed.

Given:

$P_s = 7357.7$ kw

$H = 5.5$ m

$u_1 = K_u \sqrt{2gH}$ $\qquad\qquad$ $k_f = 2.68$

$= 2.09 \sqrt{2g \times 5.5}$ $\qquad\qquad$ $K_u = 2.09$

Solution

$D_b = \frac{1}{3} d_o$

$u_1 = 21.7$ m/sec

D_o = Boss dia

$k_f \sqrt{2gH} = V_{f1}$

D_h = hub dia

$V_{f1} = 2.68 \times \sqrt{2gH}$

$V_{f1} = 27.064$ m/sec

$$\eta_o = \frac{P_s}{\text{Water power}} = \frac{7357.7}{WQH}$$

$Q = 227.27$ m^3/sec.

$$Q = \frac{\pi}{4}(D_o^2 - D_h^2) \times V_{f1}$$

$$= \pi/4 \left(D_o^2 - \left(\tfrac{1}{3}(D_o^2)\right)\right] \times 7.064.$$

$D_o = 6.788$ m.

$$D_b = \frac{1}{3} D_0 = 2.26m.$$

$$u_1 = \frac{\pi D_0 N}{60} \quad ; \quad N = 61.08 \text{ rpm}.$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = 622 \text{ rpm}.$$

problem (3)

A pelton wheel is to be designed for the following specifications.

Shaft power = 11772 kw

Head = 380m

speed = 750 rpm

η_0 = 86% , jet diameter is not exceed one-sixth of wheel diameter. Determine,

i) The wheel diameter

ii) The no. of jets required

iii) Diameter of jet

Take $k_{v_1} = 0.985$; $k_{u_1} = 0.45$

Given:

$S.P = 11772$ kw

$H = 380$ m

$N = 750$ rpm

$\eta_0 = 86\%$

$$d = \frac{1}{6} \times D, \quad \frac{d}{D} = \frac{1}{6}$$

Co-eff of velocity $k_{v_1} = C_v = 0.985$

Speed ratio $k_{u_1} = 0.45$

Solution:

velocity of jet $v_1 = C_v \sqrt{2gH}$

$= 0.985 \sqrt{2 \times 9.81 \times 380}$

$= 85.05$ m/sec.

$u = u_1 = u_2$

$u = K_u \sqrt{2gH} = 0.45 \sqrt{2 \times 9.81 \times 380}$

$= 38.85$

$u = \frac{\pi D N}{60} \Rightarrow 38.85 = \frac{\pi D N}{60}$

$D = 0.989$ m.

$d/D = 1/6$; $d = 0.16$ m.

Discharge of jet q = Area of jet × velocity of jet

$= \pi/4 \times d_1^2 \times v_1$

$= 1.818$ m³/sec.

$\eta_o = \frac{sp}{wp} = \frac{11772}{wQH}$

Total Discharge $\cancel{} \times w \times H \times Q = \frac{11772}{0.86}$

$9.81 \times 380 \times Q$ = Ans

$Q = 3.672$ m³/sec.

find,

Number of jet $= Q/q = \frac{3.672}{1.818}$

$= 2$ jets

(4) The penstock supplies water from a reservoir to the pelton turbine with an gross head of 500m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m³/sec. The angle of deflection of the jet is 165°. Determine the power given by the water to runner and also hyd. efficiency of the pelton wheel. Take speed ratio = 0.45 and $C_V = 1.0$

Given:

$H_g = 500m$.

Head loss in friction $h_f = \frac{H_g}{3} = \frac{500}{3}$

$= 166.7m$.

Net head $H = H_g - h_f = 500 - 166.7 = 333.3m$.

$Q = 2.0 m^3/sec$.

Angle of deflection = 165°

$\phi = 180° - 165 = 15°$.

Speed ratio $K_u = 0.45$

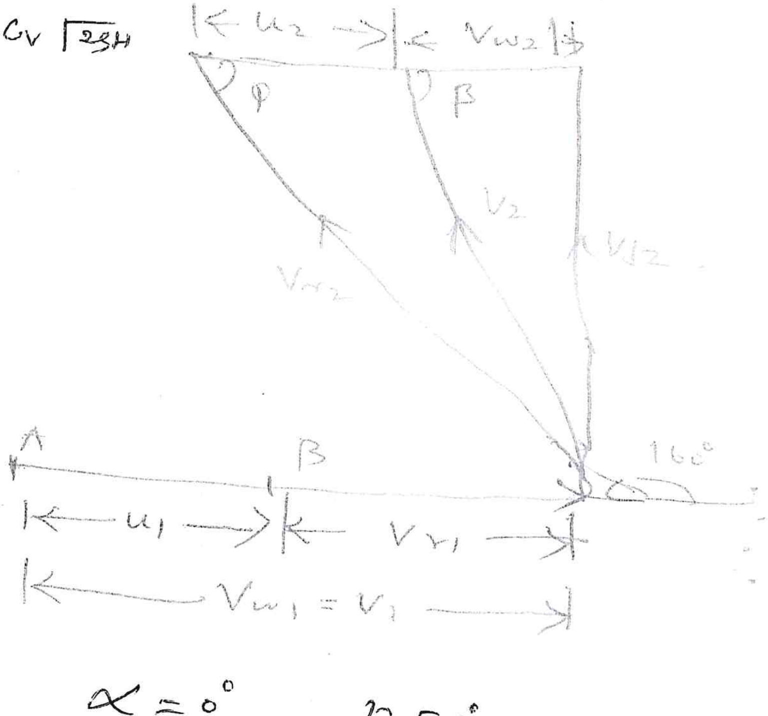
Solution:

Vel. of jet $V_1 = C_V\sqrt{2gH}$

$V_1 = 80.86 m/sec$.

$u_1 = K_u\sqrt{2gH}$

$= 36.387 m/sec$.



$V_{r_1} = V_1 - u_1$

$= 80.86 - 36.387$

$= 44.47 m/sec$.

$V_{w_1} = V_1 = 80.86$.

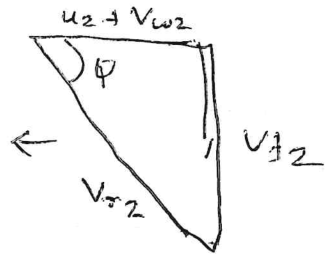
$V_{w_1} = V_1$

$\alpha = 0°$, $\theta = 0°$.

From Triangle.

$$V_{r_2} = V_{r_1} = 44.478 \text{ m/sec}$$

$$V_{r_2} \cos\phi = u_2 + V_{w_2}$$



$$V_{w_2} = 6.57 \text{ m/sec}$$

workdone, $\rho a v_1 \left[V_{w_1} + V_{w_2}\right] \times u$

$$\rho . Q \left[V_{w_1} + V_{w_2}\right] \times u$$

$$1000 \times 2 \times \left[80 + 6.5\right] \times 36$$

$$= 6362680 \text{ N-m/sec.}$$

$$\text{Power} = \frac{\text{workdone}}{1000} = \frac{6362680}{1000}$$

$$= 6362.63 \text{ k.w.}$$

$$\eta_{hyd} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = 97.3\%$$

Note:

1. semi circle buckets $\phi = 0°$.
2. clearance angle ϕ
3. Jet ratio $m = D/d$.
4. No. of buckets $Z = \frac{D}{2d} + 15$
5) mean velocity / bucket speed $= u$.
6. Turbine, Discharge $Q_u = \frac{Q}{\sqrt{H}}$.
7. Unit speed $N_u = \frac{N}{\sqrt{H}}$
8. Unit Power $P_u = \frac{P}{H^{3/2}}$. $\eta_{hyd} = \frac{1}{2} m . v^2$.

⑤ An inward flow reaction turbine develops 260 HP at an overall efficiency of 78% under a head of 70 m. The peripheral speed of vanes at inlet is 35 m/sec. Width of wheel at inlet is one-sixth of the corresponding diameter. Velocity of flow remains constant at 5 m/sec. Outlet diameter of vanes is three-fourth of inlet diameter. If inlet angle of runner vane is 90° to the tangent, determine the guide blade angle and runner vane outlet angle. Velocity of whirl at outlet is zero

Given:

$$\text{power } P = 260\,HP$$
$$= 260 \times 0.746 = 193.96 \text{ kw}$$

$$\eta_0 = 78\% = 0.78$$

$$\text{Head } H = 70 m$$

Velocity of wheel $u_1 = 35\ m/s$

width of wheel at inlet $B_1 = 1/6$ of diameter at inlet $= D_1/6$

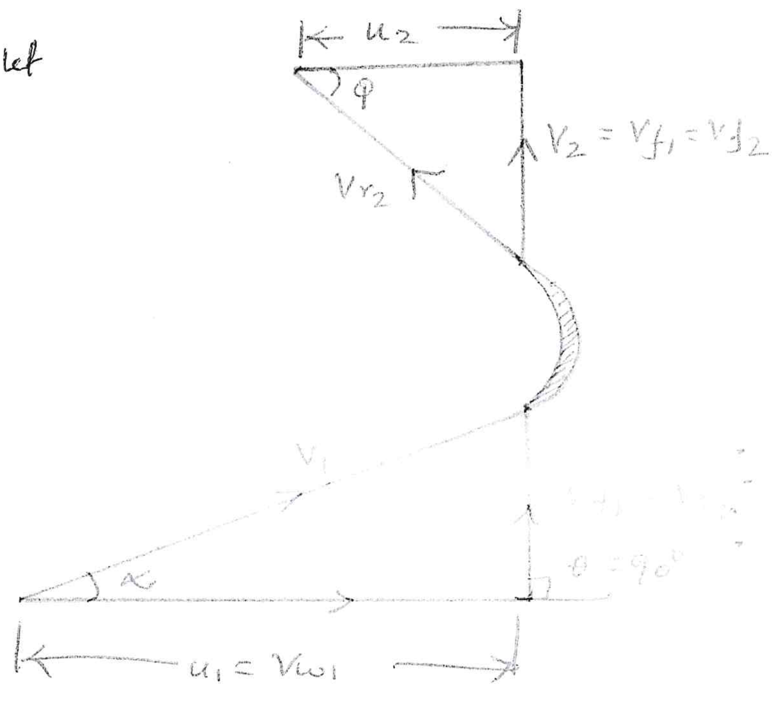
outer diameter of blade $D_2 = 3/4$ of inlet diamter $= 3D_1/4$.

Runner vane angle at inlet $\theta = 90°$

$$V_{w_2} = 0$$

Solution:

Runners vanes are radial at inlet



$$\theta = 90°$$

$$V_{w_1} = u_1$$

$$V_{w_2} = 0 ; \ V_2 = V_{f_2} = 5\ m/sec.$$

From inlet velocity triangle

$$u_1 = V_{w_1} = 35\ m/sec.$$

$$V_{r_1} = V_{f_1} = 5\ m/sec.$$

overall efficiency $\eta_0 = \dfrac{P}{wQH}$

$$0.78 = \frac{193.96}{9.81 \times Q \times 70}$$

$$Q = 0.362 \ m^3/sec.$$

w.k.T,

$$Q = \pi D_1 B_1 V_{f1}$$

$$0.362 = \pi \times D_1 \times (D_1/6) \times 5$$

$$D_1 = 0.37185 = 371.8 \ mm$$

Outer diameter of blade $D_2 = \cancel{3/D_1}\ 3D_1/4$

$$= 3/4 \times 371.85$$

$$= 279 \ mm.$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$35 = \pi \times 0.37185 \times N/60$$

$$N = 1797 \ rpm$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.27889 \times 1797}{60}$$

$$u_2 = 26.25 \ m/sec.$$

From outlet velocity Triangle

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{5}{26.25}$$

$$\phi = 10^\circ \ 7'$$

From, outlet $\beta = 90^\circ$ $(V_{w1} = u_1)$

$$\tan \alpha = \frac{V_{f1}}{u_1} = \frac{5}{35}$$

$$\alpha = 8.13^\circ$$

Unit - V

Pumps

Classification of pumps

A pump is a device which converts the mechanical Energy supplied into hydraulic energy by lifting water to higher level.

The lifting of water to a higher level is carried out by the various action of pumps such as centrifugal action, reciprocating action etc.

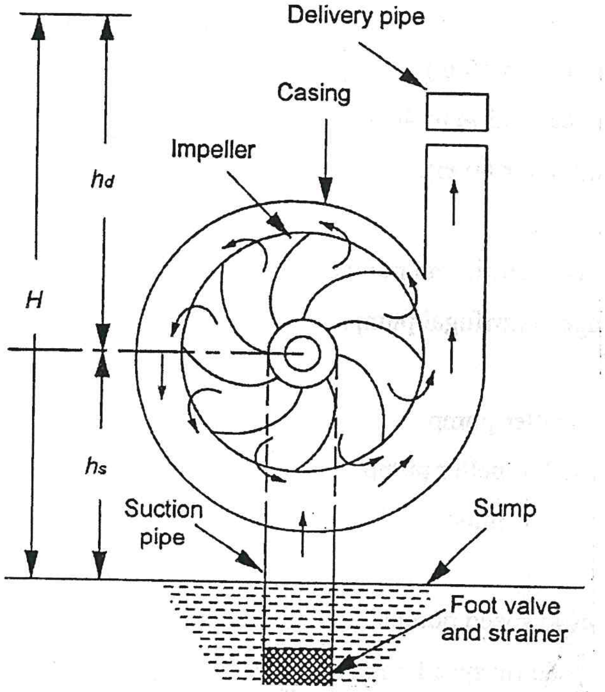
The hydraulic energy refers both potential and kinetic energies of liquid. Hydraulic pumps are the energy absorbing machines.

Pumps

- Rotodynamic pumps
  - Radial flow pump
  - Axial flow pump
  - mixed flow pump
- Positive displacement pum[p]
  - Reciprocating Pump
    1. Piston type
    2. Diaphram type
  - Rotary Pump
    1. Gear type
    2. Vane typ[e]
    3. Screw type.

Centrifugal pumps

This type of device convert the mechanical energy into hydraulic energy by means of the centrifugal force acting on the fluid.



The main parts centrifugal pumps are.

1. Impeller - the rotating element of vane rotor
2. Casing - It is an airtight chamber surrounding the impeller
3. Suction pipe - Transfer the fluid from sump to pump
4. Delivery pipe - Transfer the fluid ~~from~~ pump to head (H)
5. Strainer - arrest the debris in the sump.
6. Foot valve - It is a one directional valve & donot allow the water in return flow.

Working of pump

i) priming : The suction pipe, casing and delivery pipe are filled with the liquid for no air. This is called priming.

ii) After that priming, through the electric motor the fluid lifted from suction to delivery by the helping of impeller.

Working principle

Step I.

The delivery valve is closed. The suction pipe, Casing and portion of the delivery pipe upto the delivery valve are completely filled with the liquid so that no air pocket is left. This process is called priming.

Step II.

The electric motor is Started to rotate the impeller by keeping the delivery valve still closed. The rotation of the impeller causes strong suction or vacuum just at the eye of the casing.

Step III. The speed of the impeller is gradually increased till the impeller rotates at its normal speed and it develops normal hydraulic energy required for pumping the liquid.

Step IV

Now the liquid continuously sucked by the suction pipe and it passes through the eye of the casing.

step - V

Then the liquid is enter to casing and the continuous of centrifugal force it will transferred to the delivery pipe.

Step - VI

When the pump is stopped, the delivery pipe's valves should be closed. If there is a foot valve, no need of closing delivery valve (one direction flow).

various heads of a pump

1. Suction head: (h_s) : It is the vertical height of the centre line of the pump shaft above the liquid surface in the sump from which the liquid is being rised.

2. Delivery head (h_d):
It is the vertical height of the liquid surface in the tank to which the liquid is delivered above the centre line of the pump shaft.

3. Static head (H_{st}):
The sum of suction and delivery head is known as static head.

$$H_{stat} = h_s + h_d.$$

4. Manometric head (H_m): It is the head against which a centrifugal pump has to work. It is given by the following expressions.

$$H_m = \frac{V_{w2} u_2}{g} - (h_{Li} + h_{Lc}).$$

h_{Li} - loss of head in impeller
h_{Lc} - Loss of head in casing.

$$H_m = H_{stat} + \underset{(h_{fs} + h_{fd})}{\text{loss of pipes}} + \frac{V_d^2}{2g}.$$

$$H_m = \left(\frac{P_2}{w} + \frac{V_1^2}{2g} + Z_2\right) - \left(\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1\right)$$

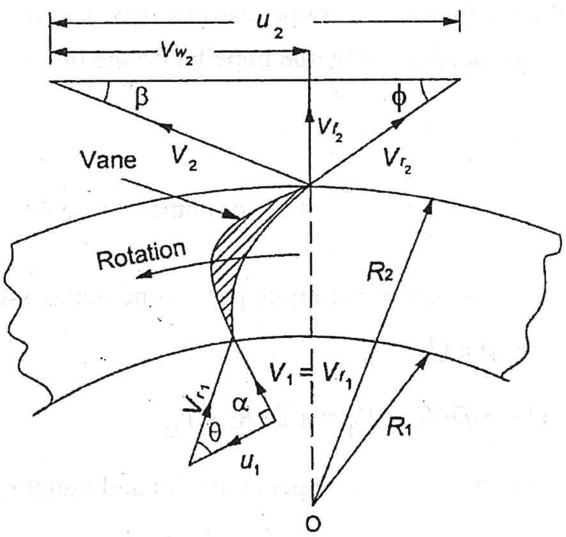
overall efficiency

It is the ratio between the input and output power of the pump.

$$\eta_{overall} = \frac{\text{output Power}}{\text{Input power}} \times 100$$

Velocity Triangle

The below figure shows the vane of the impeller with inlet and outlet velocity triangles.



Here,

N - speed of the impeller

D_1 - Diameter of the impeller

u_1 = velocity of impeller at inlet ; V_{f_1} = velocity of flow - inlet

V_1 = Absolute velocity of water at inlet

V_{w_1} = velocity of whirl at inlet

V_{r_1} = Relative velocity of liquid at inlet ~~by V_1~~

α - vane angle at inlet by V_1

θ - Angle made by V_{r_1} in vane at inlet.

And the D_2, V_2, V_{r_2}, V_{w_2}, V_{f_2} & φ are the outlet values.

The workdone/sec by the liquid on the runner

$$\frac{W}{g}\left(V_{w_1} u_1 \pm V_{w_2} u_2\right)$$

Here W - weight of liquid/sec passes through the impeller

$W = \rho \cdot g \cdot Q$.

Discharge $Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 B_2 \times V_{f_2}$.

workdone of impeller/sec.

$$= \frac{W}{g}\left(V_{w_2} u_2\right)$$

; $\alpha = 90^\circ$

$\therefore V_{w_1} = 0$

Workdone by impeller

$$\text{Manometric head } (H_m) = \frac{V_{w2}\, u_2}{g} - (h_{Li} + h_{Lc})$$

h_{Li} - loss of head in impeller
h_{Lc} - loss of head in casing.

Efficiency of pump

i) $$\eta_{mano} = \frac{H_{mano}}{\text{Head imparted by impeller}} = \frac{H_m}{V_{w2}\, u_2 / g}$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \cdot u_2}$$

ii) Vol. efficiency : $$\eta_{vol} = \frac{Q_{pump}}{Q_t}$$

iii) Mechanical efficiency :

$$\eta_{mech} = \frac{\text{Power at Impeller}}{\text{Power at shaft}} = \frac{\frac{W}{g}\left(\frac{V_2\, u_2}{1000}\right)}{P_{shaft}}.$$

iv) output efficiency :

$$\eta_o = \frac{\text{Output Power}}{\text{input power}} = \frac{WQH_m}{P}$$

v) Specific speed (N_s)

$$N_s = \frac{N\sqrt{P}}{H_m^{5/4}}.$$

Cavitation

The harmful effects of cavitations are follows,

i) pitting and erosion of surface
ii) sudden drop head
iii) noise and vibrations.

Performance curves

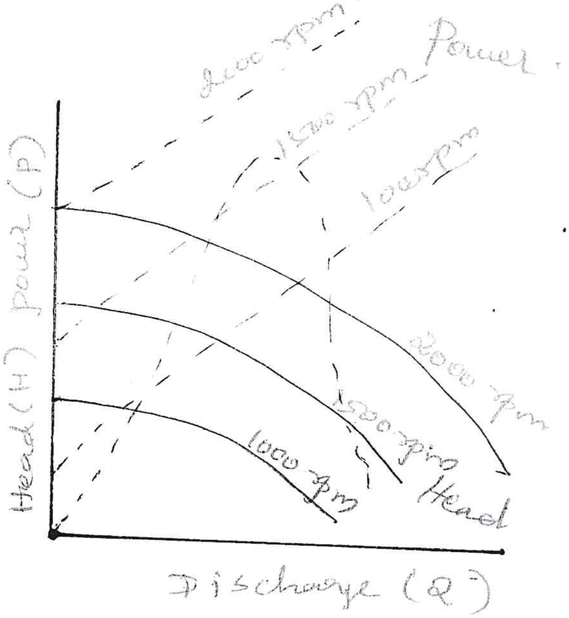
The curves which are plotted from the series of a number of tests on the centrifugal pump are known as performance curves.

It refers the graphical representation of a variation in head, power and efficiency of pump drawn to a common of flow rate.

The purpose of these curves is to predict the behaviour and performance of a pump under varying conditions.

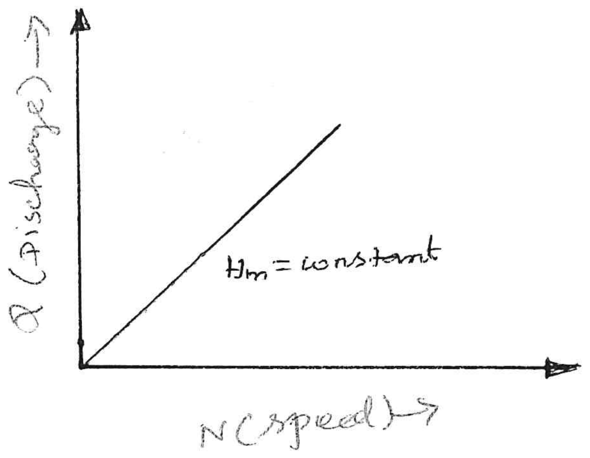
To obtain main characteristics of a pump, it is run at a constant speed and the discharge is varied over the range.

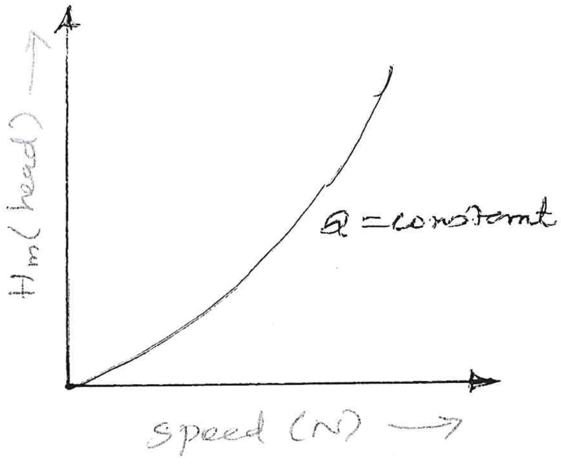
The curves are plotted for head, power and efficiency against discharge.



Main characteristic curve of pump.

Constant Discharge & constant head curves





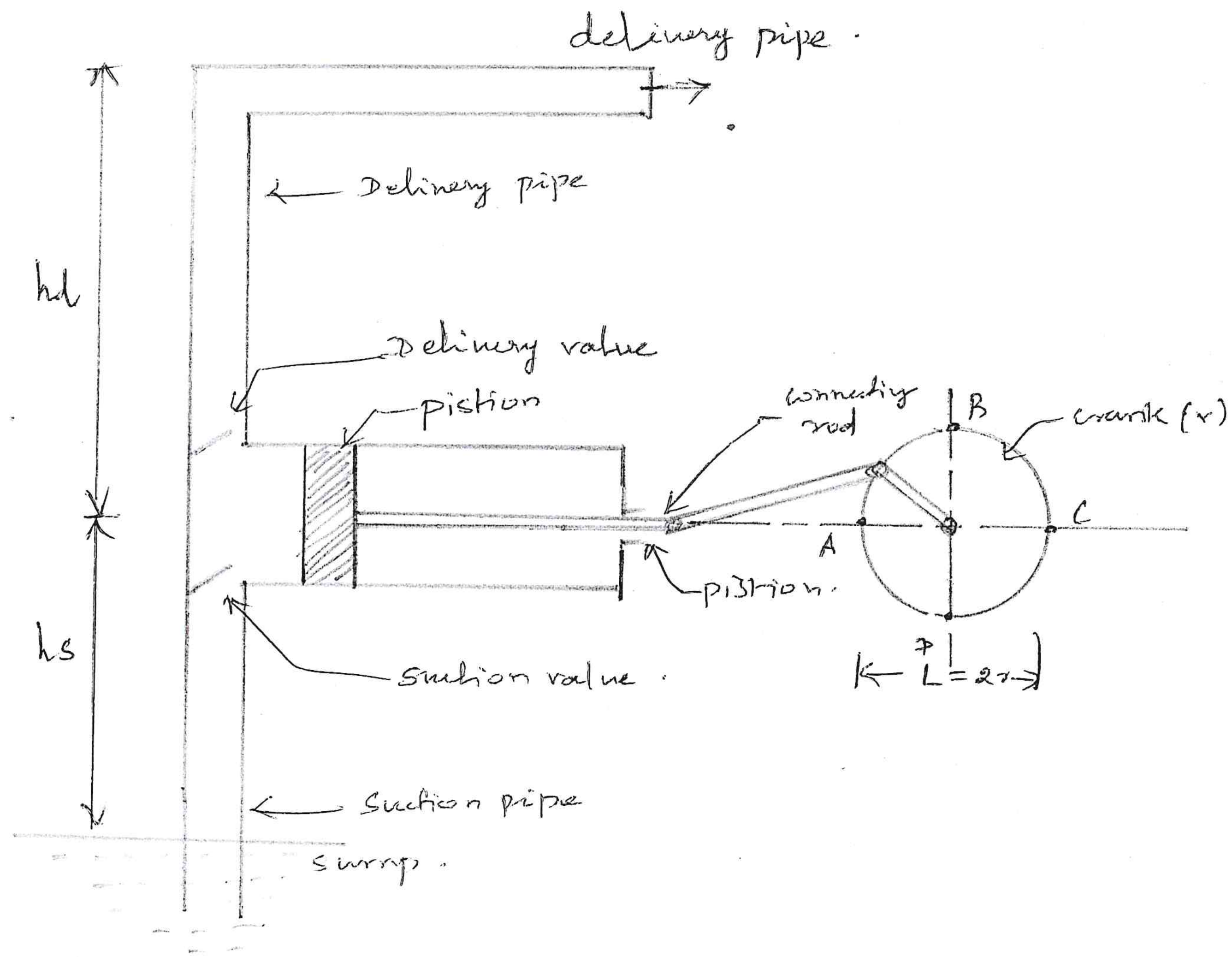
The performance of a variable speed pump is for which the constant speed variation can be obtained by these curves. When the pump has a variable speed, the plotted graph b/w Q and N, and Hm, N may be obtained.

Reciprocating pump

The hydraulic machines which convert the mechanical energy into hydraulic energy which is mainly in the form of pressure energy.

If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy, the pump is known as Reciprocating pump.

Construction:



1. The cylinder with pistion
2. Suction pipe & Delivery pipe
3. Suction valve & Delivery valve.
4. Connecting rod & Crank.

Working principle

i) The single acting Reciprocating pump, which consists of a pistion which moves forward and backward in a close fitting cylinder. The movement of the pistion is obtained by the connecting rod.

ii) The crank is rotated by the electric motor. The Suction and delivery pipes with suction and delivery values are connected to the cylinder. The two values has allows the one direction of fluid to the system.

iii) when the crank starts rotating, the piston moves in the cylinder to the leftward of the first half cycle. In this time the suction value in closed contion and the delivery value is open condition. so the water is discharge to exit through the delivery pipe.

iv) when the crank rotating from (180 - 0°), the pistion moves in the cylinder to the rightward of the second half cycle. In this time the sution value in open condition and the delivery value is closed condition. so the water fullfill the inside of the cylinder through the sution pipe.

Discharge of pump

$$Q = \text{Area} \times \text{length} \times N/60$$

$$= A \times L \times \frac{N}{60}$$

$$Q = ALN/60$$

weight of water is delivery per second

$$W = \rho \times g \times Q$$

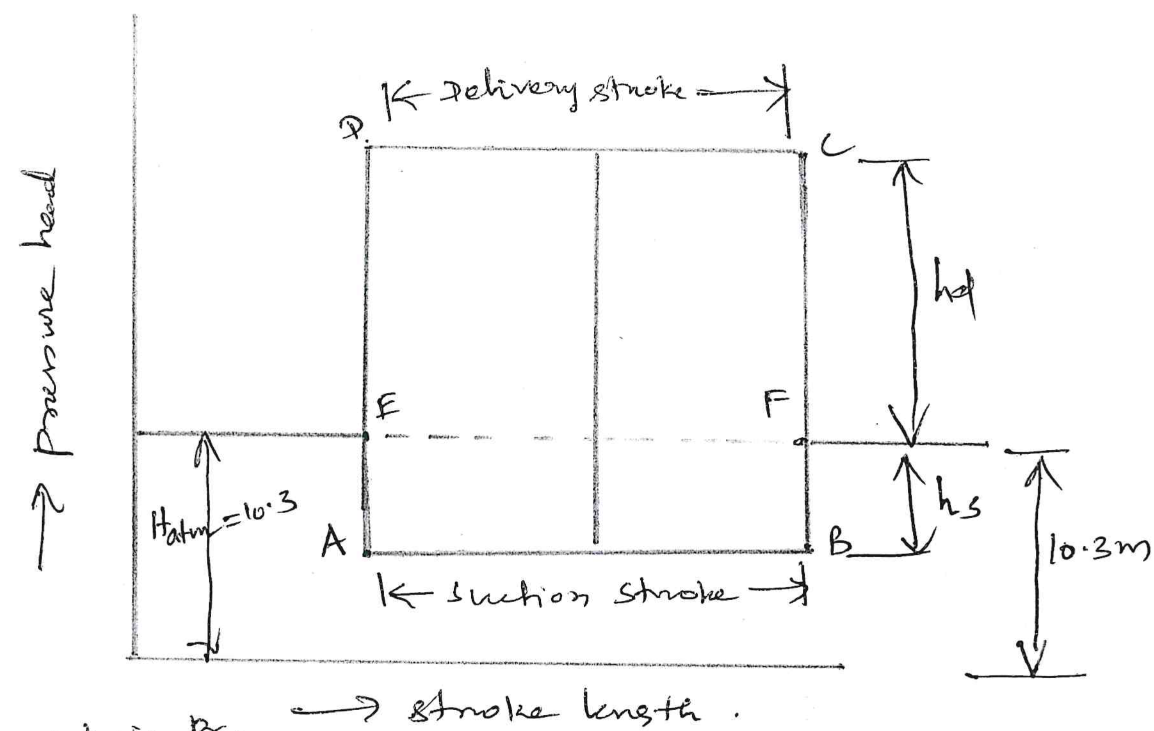
$$= \frac{\rho \times g \times ALN}{60} = \frac{\rho \cdot g \cdot A \cdot L \cdot N}{60}$$

$$\text{work done} = \frac{\rho g \cdot ALN \cdot \times (hs + hd)}{60 \times 1000} \text{ kw}.$$

Indicator diagram Read.

This diagram is defined as the graph between the pressure head in the cylinder and the distance travelled by pistion from inner dead centre for one complete revolution of the crank.

As the maximum distance travelled by the pistion is equal to the stroke length and hence the indicator diagram is a graph b/w pressure head and stroke length of the pistion for one complete revolution.



Here,

H_{atm} - Atmospheric Pr. head.

L - Length of stroke

h_s - Suction head.

h_d - delivery head.

$$k = \frac{\rho g A N}{60}$$

w.k.T, Workdone of pump $W = \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$.

$$= k \times L (h_s + h_d).$$

$$\propto L \times (h_s + h_d).$$

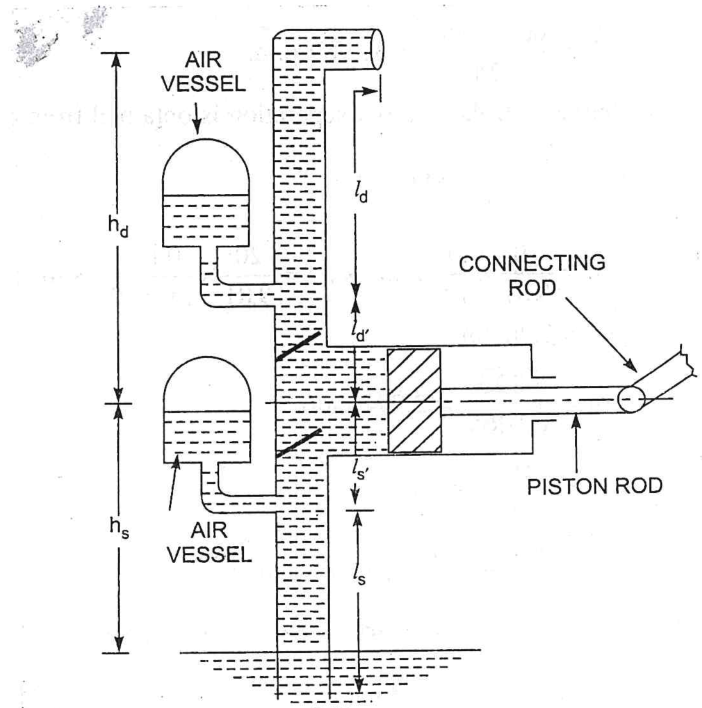
Area of indicator diagram

$$= AB \times BC = AB \times (BF + FC)$$

$$= L \times (h_s + h_d)$$

Work Saved by Fitting Air vessels

Air vessels are closed chamber containing compressed air in the top portion and liquid at the bottom of the chamber. When the liquid enters the air vessel, the air will expand in the chamber. The air vessel fitted to the suction pipe and delivery pipe at a point close to the cylinder of the reciprocating pump.



The above figure shows the arrangement of air vessel in single acting reciprocating pump

Workdone saved per $= W_1 - W_2$

$$= \frac{2}{3} L \times \frac{4fL}{2g.d} \times \left(\frac{A}{a} \omega r\right)^2 - \frac{1}{\pi^2} L \times \frac{4fL}{2gd} \times \left(\frac{A}{a} \omega r\right.$$

$$= L \times \frac{4fL}{2gd} \times \left(\frac{A}{a} \omega r\right)^2 \left(\frac{2}{3} - \frac{1}{\pi^2}\right)$$

The percentage of the work saved per stroke.

$$= \left(\frac{W_1 - W_2}{W_1}\right) \times 100 = \frac{L \times \frac{4fL}{2gd} \left(\frac{A}{a} \omega r\right)^2 \left(\frac{2}{3} - \frac{1}{\pi^2}\right)}{\frac{2}{3} L \times \frac{4fL}{2gd} \times \left(\frac{A}{a} \omega r\right)^2} \times 100$$

$$= \left[\left(\frac{2}{3}\right) - \left(\frac{1}{\pi^2}\right) / \left(\frac{2}{3}\right)\right] \times 100$$

Rotary Pumps Regal

It is a rotary Pump in which two gears mesh to provide the pumping action. This type of pump is mostly used for cooling water and pressure oil to be supplied for lubrication to motors, turbines and m/c tool.

Here the flow of liquid to be pumped is continuous and uniform.

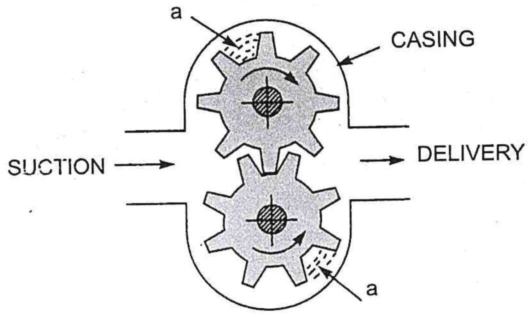


Fig. 21.14 *Gear-wheel pump.*

Construction:

i) Figure shows the gear pump, which consists of two intermeshing gears working in a fine clearance inside a casing.

ii) one of the gear is keyed to a driving shaft. The other gear revolves due to driving gear.

iii) The space between teeth and the casing is filled with oil. The oil is carried round between the gears from the suction pipe to the delivery pipe.

iv) The oil pushed into the delivery pipe, cannot back into the suction pipe due to meshing of the gears.

Here,

Volume of oil Discharged per revolution $= 2 \times a \times L \times N \ m^3$

$= 2aLn \times \frac{N}{60} \ m^3.$

N - Speed of gear

a - inside carry area

n - Total no. of teeth

L - Axial length of teeth

$$\text{The volumetric efficiency} = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

Problem ①

The impeller of a centrifugal pump had an external diameter of 450mm and internal diameter of 200mm and it runs at 1440 rpm. Assuming a constant radial flow through the impeller at 2.5 m/sec and that the vanes at exit are set back at an angle 25°.

Determine,

i) Inlet vane angle

ii) The angle absolute velocity of water at exit makes with the tangent.

iii) The workdone /N of water

Given

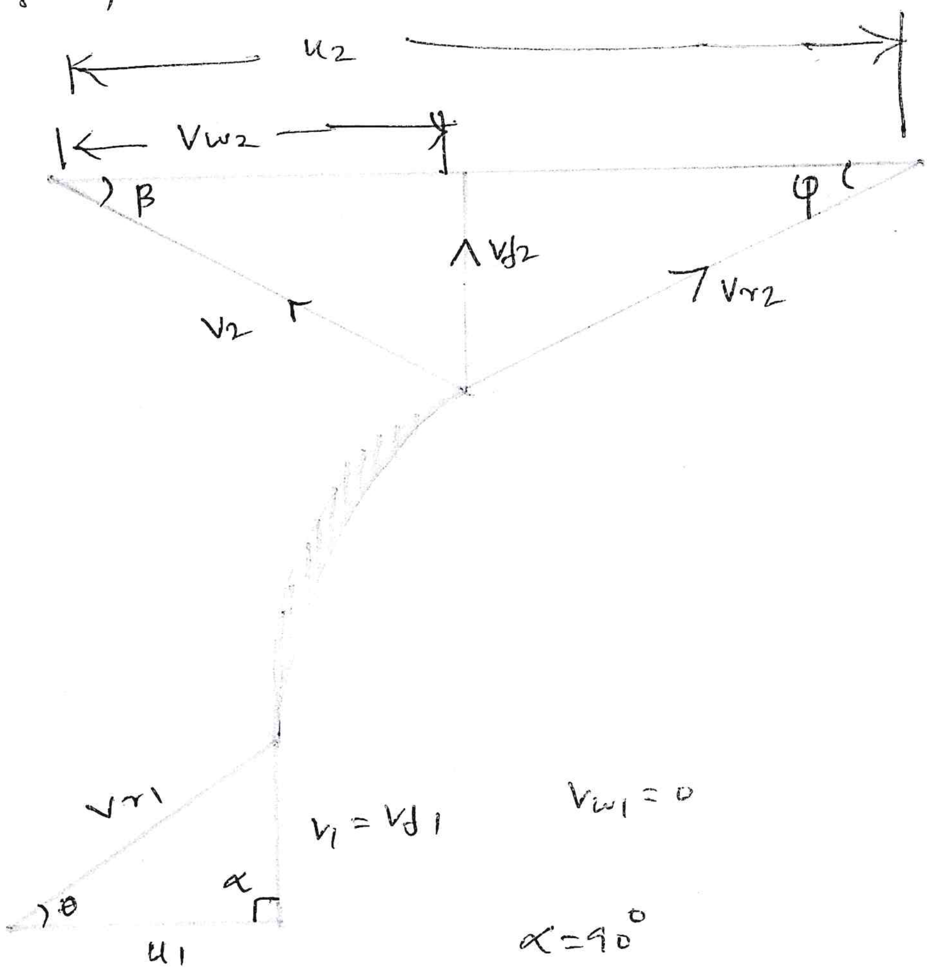
Internal dia $D_1 = 200mm$

Ext-dia $D_2 = 450mm$

$N = 1440$ rpm

$V_{f1} = V_{f2} = 2.5$ m/sec.

Vane angle $\varphi = 25°$

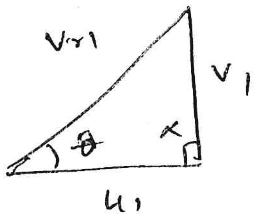


Solution:

i) Inlet vane angle (θ)

$$u_1 = \frac{\pi D_1 N}{60} = 15.08 \text{ m/sec.}$$

from vel. Triangle at inlet.



$$\tan\theta = \frac{V_{f1}}{u_1}$$

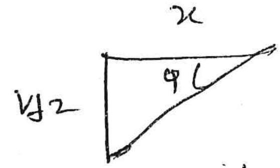
$$= \frac{2.5}{15.08}$$

$$\theta = \tan^{-1}(0.16) = 9.4^\circ$$

ii) The angle, absolute velocity of water β (outlet).

$$u_2 = \frac{\pi D_2 N}{60} = 33.93 \text{ m/se.}$$

from velocity Triangle.

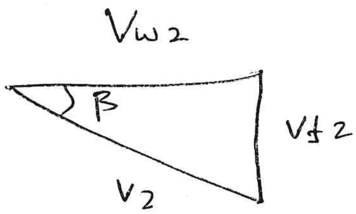


$$\tan\varphi = \frac{V_{f2}}{x}.$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\varphi}$$

$$= 33.93 - \frac{25}{\tan 25}$$

$$V_{w2} = 28.57 \text{ m/sec.}$$



So, $$\tan\beta = \frac{V_{f2}}{V_{w2}}$$

$$= \frac{2.5}{28.57}$$

$$\beta = 5^\circ$$

iii) workdone / N of water. (not consider weight).

$$= \frac{V_{w2} * u_2}{g} = \frac{28.51 \times 33.93}{9.81} = 98.81 \text{ N-m.}$$

Tangential velocity at inlet $u_1 = \pi D_1 N/60 = 18.85\ m/sec$.

" " " outlet $u_2 = \pi D_2 N/60 = 37.7\ m/sec$

From the inlet triangle

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{3}{18.85} = 0.159$$

$$\theta = \tan^{-1}(0.159) = 9^\circ 2'$$

Discharge $Q = \pi D_2 B_2 \times V_{f2}$

$= \pi \times 0.6 \times 0.05 \times 3$

$= 0.28\ m^3/sec$.

From outlet velocity triangle

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\tan 30^\circ = \frac{3}{37.7 - V_{w2}} \Rightarrow 5.2\ m/sec \Rightarrow V_{w2}$$

$$V_{w2} = 37.7 - 5.2 = 32.5\ m/sec.$$

workdone per second by the impeller

$$\frac{W}{g} V_{w2} u_2 = \frac{\rho g Q}{g} \times V_{w2} u_2$$

$$= \frac{1000 \times 9.81 \times 0.28}{9.81} \times 32.5 \times 37.7$$

$$= 343070\ W = 343.07\ kW.$$

Manometric efficiency $\eta_{mano} = \frac{g H_m}{V_{w2} \cdot u_2}$

$$= \frac{9.81 \times 32}{32.5 \times 37.7}$$

$$\eta_{mano} = 0.2562 = 25.62\%$$

$$H_{Abs} = H_{atm} - (H_s + H_{as})$$

$$= 4.67\,m.$$

b) middle of suction stroke

$\theta = 90°$.

$H_{as} = 0$.

$$H_{abs} = H_{atm} - (h_s + h_{as})$$

$$= 10.3 - 3 = 7.3\,m.$$

c) At the end of suction stroke.

$$H_{as} = \frac{L}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r \cos 180° \qquad \theta = 180°.$$

$$= -2.63\,m.$$

Pr. head $= h_s + h_{as}$.

$$= 3 - 2.63 = 0.37\,m.$$

$$H_{Abs} = H_{atm} - (h_s + h_{as})$$

$$= 10.3 - 0.37$$

$$= 9.93\,m.$$

ii) Pr. head on delivery pipe.

a) At begining $\theta = 0°$.

$$h_{ad} = \frac{L_d}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r \cos\theta$$

$$= 5.26\,m.$$

Pr. head $= h_d + h_{ad} = 20 + 5.26 = 25.26\,m$.

$P_{abs} = H_{atm} + (h_d + h_{ad}) = 35.56\,m$.

b) middle $\theta = 90^\circ$

$h_{ad} = 0$

$H_{abs} = H_{atm} + h_d = 30.3\,m.$

c) End of delivery stroke

$\theta = 180^\circ$

$= -5.26\,m$

$H_{abs} = 25.04\,m$

ii) Power $P = w \times Q \times H_{Total}.$

$= 9.81 \times \frac{ALN}{60} \times (3 + 20).$

$= 446.7 \times 10^3\,W.$

Note:

Both $(h_{as} + h_{fs})$.

$Pr.\ head = h_s + h_{as} + h_{fs}$ (suction).

$Pr.\ head = h_d + h_{ad} + h_{fd}$ (delivery)

$P_{abs} = H_{atm} - (P_s + P_{as} + P_{ss})$ (−) (suction)

$P_{abs} = H_{atm} + (P_d + P_{ad} + P_{sd})$ (+) (delivery)

(3) A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200 rpm. works against a total head of 32 m. The velocity of flow through the impeller is constant an equal to 3 m/sec. The vanes are set back at an angle of 30° at the outlet. If the outer diameter of the impeller is 600 mm and width at outlet is 50 mm, Determine the.

a) vane angle at inlet
b) workdone per second by impeller
c) manometric efficiency.

Given data

$D_2 = 2D_1$

speed $N = 1200$ rpm

Head $H_m = 32$ mm

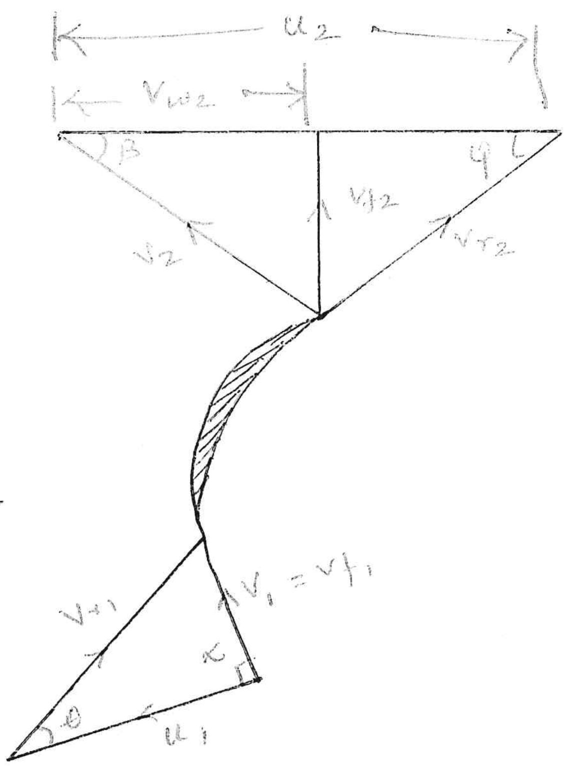
velocity of flow $V_{f1} = V_{f2} = 3$ m/sec

vane angle at outlet $\phi = 30^\circ$

outer diameter of impeller $D_2 = 600$ mm $= 0.6$ m.

width at outlet $B_2 = 50$ mm $= 0.05$ m

Solution



Inner diameter of impeller

$$D_1 = \frac{D_2}{2} = \frac{0.6}{2} = 0.3 \text{ m}$$

Tangential velocity at inlet

$$u_1 = \frac{\pi D_1 N_1}{60} = \frac{\pi \times 0.3 \times 1200}{60}$$

$$u_1 = 18.85 \text{ m/sec.}$$

Tangential velocity at inlet $u_1 = \pi D N/60 = 18.85$ m/sec.

" " " outlet $u_2 = \pi D_2 N/60 = 37.7$ m/sec

From the inlet triangle

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{3}{18.85} = 0.159$$

$$\theta = \tan^{-1}(0.159) = 9^\circ 2'$$

Discharge $Q = \pi D_2 B_2 \times V_{f2}$

$= \pi \times 0.6 \times 0.05 \times 3$

$= 0.28\ m^3/sec.$

From outlet velocity triangle

$$\tan\varphi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\tan 30^\circ = \frac{3}{37.7 - V_{w2}} \Rightarrow 5.2\ m/sec \Rightarrow V_{w2}$$

$$V_{w2} = 37.7 - 5.2 = 32.5\ m/sec.$$

workdone per second by the impeller

$$\frac{W}{g} V_{w2} u_2 = \frac{\rho g Q}{g} \times V_{w2} u_2$$

$$= \frac{1000 \times 9.81 \times 0.28}{9.81} \times 32.5 \times 37.7$$

$= 343070\ W = 343.07\ KW.$

Manometric efficiency $\eta_{mano} = \frac{g H_m}{V_{w2} \cdot u_2}$

$$= \frac{9.81 \times 32}{32.5 \times 37.7}$$

$\eta_{mano} = 0.2562 = 25.62\%$

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